① $\cot\alpha = \frac{\cos\alpha}{\sqrt{\frac{1-\cos^2\alpha}{\sin^2\alpha}}}$ ، $\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$

$\frac{1}{|\cos\alpha|} - \tan\alpha = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \tan\alpha = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \cos\alpha > 0$ (I)

$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha > 0$ (II)

(I) و (II) ∈ ناحیه اول

②

$\sin 2x = \frac{m-1}{4}$ $\frac{-\pi}{12} < x < \frac{3\pi}{12}$

$\frac{-\pi}{12} < x < \frac{3\pi}{12}$ ×۲ → $\frac{-\pi}{6} < 2x < \frac{3\pi}{6}$

$\frac{-1}{2} < \frac{m-1}{4} \leq 1 \Rightarrow -2 < m-1 \leq 4 \rightarrow \boxed{-1 < m \leq 5}$

③

$2\pi < 4x < 4\pi$ $\tan x + \cot^4 x = -3$

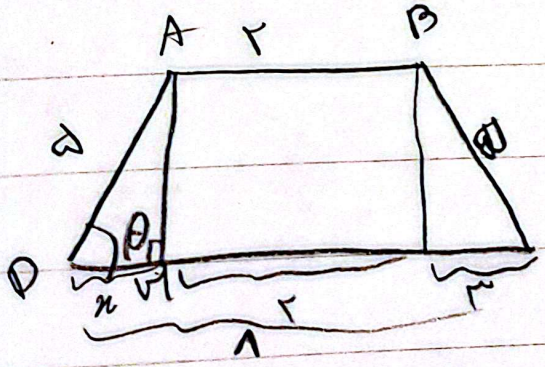
$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = -3 \Rightarrow \cos x \sin x = -\frac{1}{3}$

$2\pi < 4x < 4\pi \Rightarrow \frac{\pi}{2} < x < \pi$ ناحیه دوم → $|\sin x| < |\cos x|$

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x)} = \frac{1}{\frac{1}{\sqrt{2}} \times \frac{4}{3}} = \frac{-3\sqrt{2}}{4}$

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x \cos x = \frac{1}{3} \Rightarrow \sin x + \cos x = \frac{1}{\sqrt{3}}$

④



$\cos\theta = 0/6$

$\Rightarrow \frac{DM}{DA} = \frac{6}{10} = \frac{3}{5} \Rightarrow DM = 3$

$AH = \sqrt{25-9} = 4$

$S = \frac{(x+8)\times 4}{2} = 20$

$\tan(\text{۲۸۰}^\circ)\tan(-\text{۱۶۵}^\circ) - \sin(\text{۱۹۵})\cos(\text{۲۵۵}^\circ) = k\cos^2\text{۱۵}$ ④

$= \tan(\text{۲۷۰}^\circ + \text{۱۵})\tan(\text{۱۹۰} - \text{۱۸۰}) - \sin(\text{۱۰۸۰} + \text{۱۵})\cos(\text{۲۷۰} - \text{۱۵})$

$= (-\cot\text{۱۵})(\tan\text{۱۵}) - (\sin\text{۱۵})(-\sin\text{۱۵})$

$= \underbrace{-\text{۱} + \sin^2\text{۱۵}}_{-\cos^2\text{۱۵}} = k\cos^2\text{۱۵} \Rightarrow \underline{k = -\text{۱}}$

⑤

$A = \sqrt{\text{۳}}\cos(\text{۲۱۰})\cdot\sin(\text{۲۴۳}) - \sqrt{\text{۲}}\sin(\text{۱۳۵}^\circ)\cos(\text{۱۴۳}^\circ)$

$\sqrt{\text{۳}}\cos(\text{۱۸۰} + \text{۳۰})\sin(\text{۲۷۰} - \text{۲۷}) - \sqrt{\text{۲}}(\sin(\text{۹۰} + \text{۴۵})\cos(\text{۱۸۰} - \text{۳۷})$

$= \left(\sqrt{\text{۳}}(-\cos\text{۳۰})(-\cos\text{۲۷})\right) - \sqrt{\text{۲}}(\sin\text{۴۵})(-\cos\text{۲۷})$

$= \sqrt{\text{۳}}\left(\frac{-\sqrt{\text{۳}}}{\text{۲}}\right)(-\cos\text{۲۷}) + \sqrt{\text{۲}}\times\frac{\sqrt{\text{۲}}}{\text{۲}}(\cos\text{۲۷}) =$

$(\cos\text{۲۷})\left(\frac{\text{۳}}{\text{۲}} + \frac{\text{۱}}{\text{۲}}\right) = \frac{\text{۵}}{\text{۲}}(\cos\text{۲۷}) \Rightarrow$ برابر $\frac{\text{۵}}{\text{۲}}$

⑥

$f(x) = \text{۱۶}\cos^2(\text{۳}x)\cos^2(\text{۶}x)\cos^2(\text{۱۲}x)\cos^2(\text{۲۴}x)$

$f\left(\frac{\pi}{\text{۶}}\right) = \text{۱۶}\cos^2\left(\frac{\pi}{\text{۱۶}}\right)\cos^2\left(\frac{\pi}{\text{۶}}\right)\cos^2(\pi)\cos^2\left(\frac{\text{۴}\pi}{\text{۳}}\right)$

$\cos^2\frac{\pi}{\text{۱۶}} = \frac{\text{۱} + \cos\frac{\pi}{\text{۸}}}{\text{۲}} = \frac{\text{۱} + \frac{\sqrt{\text{۲}}}{\text{۲}}}{\text{۲}} = \frac{\text{۲} + \sqrt{\text{۲}}}{\text{۴}}$

$\Rightarrow \left(\text{۱۶}\times\frac{\text{۲} + \sqrt{\text{۲}}}{\text{۴}}\right) \times \left(\frac{\sqrt{\text{۲}}}{\text{۲}}\right)^2 \times (-\text{۱})^2 \times \left(\frac{-\text{۱}}{\text{۲}}\right)^2$

$\frac{\text{۶} + \text{۲}\sqrt{\text{۲}}}{\text{۴}}$

(1)

$\frac{1-\sin x}{1+\sin x} = 4$ ربع سوم x

$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x} \rightarrow \frac{\frac{-3}{5}}{1-\frac{4}{5}} = \frac{\frac{-3}{5}}{\frac{1}{5}} = -3$

$\frac{1-\sin x}{1+\sin x} = 4 \Rightarrow 1-\sin x = 4+4\sin x \quad 5\sin x = -3$

$\sin x = \frac{-3}{5}$

$1-\sin^2 x = \cos^2 x \Rightarrow 1-\frac{9}{25} = \frac{16}{25} \Rightarrow \cos x = \frac{-4}{5}$

(9)

$\frac{\sin\theta}{1-\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{2}$ $\Rightarrow k=2$

$\cot\frac{\theta}{2} = \frac{1+\cos\theta}{\sin\theta} = \frac{\sin\theta}{1-\cos\theta}$

(10)

$\sin\alpha = \frac{\sqrt{2}}{10}$

$\cos\left(\frac{11\pi}{4}+\alpha\right) = \cos\left(2\pi+\frac{3\pi}{4}+\alpha\right)$

$= \cos\left(-\frac{3\pi}{4}+\alpha\right) = \cos\frac{3\pi}{4}\cos\alpha - \sin\frac{3\pi}{4}\sin\alpha$

$\Rightarrow \left(-\frac{\sqrt{2}}{2} \times \frac{-\sqrt{98}}{10}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{10}\right)$

$\frac{+14}{20} - \frac{2}{20}$

$= \frac{12}{20} = \frac{3}{5}$