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یازدهم سه شنبه دفتر

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سوال ۱)

$$\cot x = \frac{\cos x}{\sqrt{1-\cos^2 x}} \Rightarrow \frac{\cos x}{\sin x} = \frac{\cos x}{|\sin x|} \Rightarrow \sin x > 0$$

$$\frac{1}{|\cos x|} - \frac{\sin x}{\cos x} = \frac{1-\sin x}{|\cos x|} \Rightarrow \cos x > 0$$

$\Rightarrow$ ناحیه اول

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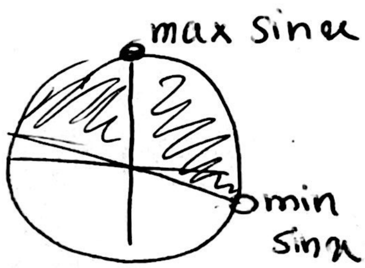
سوال ۲)

$$-\frac{\pi}{12} < x < \frac{5\pi}{12} \qquad \sin 2x = \frac{m-1}{4}$$




$$\Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow \frac{m-1}{4} \in \left(\sin -\frac{\pi}{6} , \frac{5\pi}{2}\right)$$

$$\Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \Rightarrow -2 < m-1 \leq 4$$

$-1 < m \leq 5$ یا $(-1,5]$

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سوال ۳)

$$\tan x + \cot x = \frac{1}{\sin x\cos x} = -3 \Rightarrow \sin x\cos x = \boxed{-\frac{1}{3}}$$

$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin\cos)} \Rightarrow \frac{1}{(\sin+\cos)(\frac{4}{3})}$$

$$1-\left(-\frac{1}{3}\right)$$

$$(\sin+\cos)^2 = \underbrace{\sin^2+\cos^2}_{1} + \underbrace{2\sin\cos}_{-\frac{2}{3}} \Rightarrow 1 + -\frac{2}{3} = \frac{1}{3}$$

$$\Rightarrow \sin+\cos = \sqrt{\frac{1}{3}} \Rightarrow \frac{1}{\sqrt{\frac{1}{3}}\times\frac{4}{3}} = \boxed{\frac{3\sqrt{3}}{4}}$$

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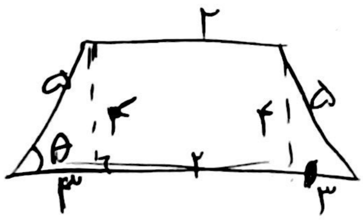
سوال ۴)




$\Rightarrow$


$$\Rightarrow \frac{4\times 10}{2} = \boxed{20}$$

سوال ۵)

$\theta = 15^\circ$ $\quad$ $1095 = \underbrace{1080}_{\text{دور کامل}} + 15 \quad$ دور کامل $\Rightarrow 360 \times 3$

$$\tan\left(\frac{3\pi}{2}+\theta\right) \times \tan(-\pi+\theta) - \sin(\theta) \times \cos\left(\frac{3\pi}{2}-\theta\right)$$

$$-\cot\theta \times \tan\theta - \sin\theta \times -\sin\theta$$

$$-1 + \sin^2\theta \Rightarrow -(\underbrace{1-\sin^2\theta}_{\cos^2\theta}) \Rightarrow \boxed{k = -1}$$

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سوال ۶)

$$A = \sqrt{3}\,\underbrace{\cos(210)}_{-\frac{\sqrt{3}}{2}}\,\sin\left(\frac{3\pi}{2}-\theta\right) - \sqrt{2}\,\underbrace{\sin(135)}_{\frac{\sqrt{2}}{2}} \times \cos(\pi-\theta)$$

$$-\frac{3}{2} \times -\cos\theta - (1 \times -\cos\theta) = \frac{3}{2}\cos\theta + \cos\theta = \frac{5}{2}\cos\theta$$

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سوال ۷)

$$16\cos^2\left(\frac{\pi}{12}\right) \times \underbrace{\cos^2\left(\frac{\pi}{6}\right)}_{\frac{3}{4}} \times \underbrace{\cos^2\left(\frac{\pi}{3}\right)}_{\frac{1}{4}} \times \underbrace{\cos^2\left(\frac{2\pi}{3}\right)}_{\frac{1}{4}}$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \quad \left(\theta = \frac{\pi}{12},\ 2\theta = \frac{\pi}{6}\right) \rightarrow \frac{1+\frac{\sqrt{3}}{2}}{2} = \frac{2+\sqrt{3}}{4}$$

$$\Rightarrow \cancel{16} \times \frac{2+\sqrt{3}}{4} \times \frac{3}{4} \times \frac{1}{\cancel{16}} = \frac{3(2+\sqrt{3})}{16} = \frac{6+3\sqrt{3}}{16}$$

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سوال ۸)

$$4 + 4\sin x = 1 - \sin x \Rightarrow 5\sin x = -3 \Rightarrow \sin x = -\frac{3}{5}$$

$$\cos x = -\frac{4}{5}$$

$$\Rightarrow \tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} \Rightarrow \sqrt{\frac{1-\cos 2a}{1+\cos 2a}} = \sqrt{\frac{1+\frac{4}{5}}{1-\frac{4}{5}}} = \sqrt{\frac{\frac{9}{5}}{\frac{1}{5}}} = 3$$

ناحیه سوم $= (180, 270) \xrightarrow{\div 2} (90, 135)$ ناحیه دوم $\Rightarrow \tan$ منفی است $\Rightarrow \boxed{-3}$

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سوال ۹) $\boxed{\text{حل بعد}}$

$$\frac{\sin(1+\cos)}{\sin^2} + \frac{1+\cos}{\sin} = \left(\frac{1+\cos}{\sin}\right) \times 2$$

$\sin^2 \to 1-\cos^2$ $\quad$ $\cos\theta \neq$ $\quad \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \leftarrow \sqrt{\frac{(1+\cos\theta)^2}{\sin^2}} \Rightarrow \left|\frac{1+\cos}{\sin}\right|$ قدر مطلق

(سوال ۹ دوباره حل شود)

سوال ۹)

$$\frac{\sin\theta}{1-\cos\theta} \rightarrow \frac{\sin\theta(1+\cos\theta)}{\frac{1-\cos^2\theta}{\sin^2\theta}} + \frac{1+\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{1+\cos\theta}{\sin} + \frac{1+\cos\theta}{\sin\theta} = 2\left(\frac{1+\cos\theta}{\sin\theta}\right)$$

$$\cot\frac{\theta}{2} = \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}} = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$$

$$\Rightarrow \left|\frac{1+\cos\theta}{\sin\theta}\right| \Rightarrow k=2$$

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سوال ۱۰) $\frac{6}{10}$ با $\frac{3}{5}$

$$\cos\left(\frac{11\pi}{4}+a\right) = \cos\frac{11\pi}{4} \times \cos\alpha - \sin\frac{11\pi}{4} \times \sin\alpha$$

$$-\frac{\sqrt{2}}{2} \times -\frac{\sqrt{98}}{10} - \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{10} = \frac{14}{20} - \frac{2}{20} \rightarrow \frac{12}{20}$$

$$* \quad 1-\left(\frac{\sqrt{2}}{10}\right)^2 = \cos^2\alpha \Rightarrow |\cos\alpha| = \frac{\sqrt{98}}{10} \xrightarrow{\text{ناحیه ۲}} -\frac{\sqrt{98}}{10}$$

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