Subject:
Date

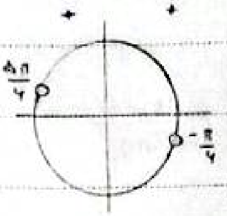
لیلا الیکی — تکلیف شماره ۲۸ — ۱۸,۷۵

۱) $\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \underset{+}{\cos\alpha} = \underset{+}{|\cos\alpha|}$ ①

$\cot\alpha = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} \Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Longrightarrow \underset{+}{\sin\alpha} = \underset{+}{|\sin\alpha|}$ ②

$\overset{1,2}{\Rightarrow}$ مثبت $\cos\alpha, \sin\alpha \Rightarrow$ ناحیه اول مثلثاتی ۵

۲) $-\frac{\pi}{12} < x < \frac{5\pi}{12} \Rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6}$



$\Rightarrow -\frac{1}{2} < \sin 2x \leq 1$

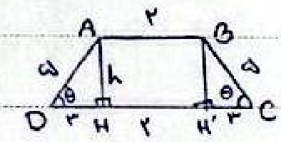
$\sin 2x = \frac{m-1}{4} \Rightarrow -\frac{1}{2} < \frac{m-1}{4} \leq 1 \Rightarrow -1 < m \leq 5$ ۵

۳) $\tan x + \cot x = \frac{2}{\sin 2x} = -3 \Rightarrow \sin 2x = -\frac{2}{3} \rightsquigarrow \sin 2x = 2\sin x\cos x = -\frac{2}{3} \rightarrow \sin x\cos x = -\frac{1}{3}$

$3\pi < 4x < 4\pi \Rightarrow \frac{3\pi}{4} < x < \pi \qquad \sin x + \cos x = \sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x} = \sqrt{1-\frac{2}{3}} = \sqrt{\frac{1}{3}}$ ۱,۷۵

$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{(\sin x\cos x)(\sin^2 x + \cos^2 x - \sin x\cos x)} = \frac{1}{(\sqrt{\frac{1}{3}})(1+\frac{1}{3})} = -\frac{3\sqrt{3}}{4}$

۴)



$\cos\theta = \frac{DH}{AD} = \frac{CH'}{BC} = \frac{6}{10} \Rightarrow DH = CH' = 3$

$AB = HH' = 2$

$\sin\theta = \sqrt{1-0.36} = 0.8 = \frac{h}{5} \Rightarrow h = 4$

$S = \frac{1}{2} \times 4 \times (2+8) = 20$ ۵

۵) $\tan(285^\circ)\tan(-165^\circ) - \sin(1095^\circ)\cos 255^\circ = \tan(\frac{3\pi}{2}+15)\tan(\pi+15) - \sin(6\pi+15)\cos(\frac{3\pi}{2}-15)$

$\Rightarrow -\cot 15\tan 15 - \sin 15(\sin 15) = \sin^2 15 - 1 = -\cos^2 15 = k\cos^2 15 \Rightarrow k = -1$ ۵

۶) $\sqrt{3}\cos 210^\circ \sin 243 - \sqrt{2}\sin 135\cos 153 = \sqrt{3} \times -\frac{\sqrt{3}}{2}\sin(\frac{3\pi}{2}-27) - \sqrt{2} \times \frac{\sqrt{2}}{2}\cos(\pi-27)$

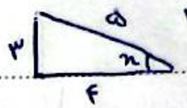
$= +\frac{3}{2}\cos 27 + \cos 27 = \frac{5}{2}\cos 27 \Rightarrow$ $\frac{5}{2}$ برابر ۲

۷) $P(\frac{\pi}{24}) = 16\cos^2(\frac{\pi}{12})\cos^2(\frac{\pi}{6})\cos^2(\frac{\pi}{3})\cos^2(\frac{2\pi}{3}) \times \frac{\sin^2(\frac{\pi}{12})}{\sin^2(\frac{\pi}{12})} \Rightarrow \frac{16 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\sin^2\frac{4\pi}{3}}{\sin^2(\frac{\pi}{12})}$

$= \frac{\sin^2(4\frac{\pi}{3})}{\frac{1-\cos\frac{\pi}{6}}{2}} = \frac{2(-\frac{\sqrt{3}}{2})^2}{1-\cos\frac{\pi}{6}} = \frac{\frac{3}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{3}{2-\sqrt{3}} = 6+3\sqrt{3}$ ۱

Subject:
Date

18) $\dfrac{1-\sin x}{1+\sin x} = 4 \Rightarrow 1-\sin x = 4 + 4\sin x \Rightarrow \sin x = \dfrac{-3}{5}$

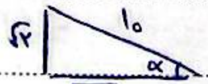


ناحیه ۳ $\Rightarrow \cos = -\dfrac{4}{5}$

$\tan \dfrac{x}{2} = \dfrac{\sin x}{1+\cos x} = \dfrac{\frac{-3}{5}}{1-\frac{4}{5}} = -3$

19) $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta} = \tan\dfrac{\theta}{2} \rightsquigarrow \dfrac{\sin\theta}{1-\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta} = \dfrac{1}{\tan\frac{\theta}{2}} + \dfrac{1}{\tan\frac{\theta}{2}} = 2\cot\dfrac{\theta}{2} \Rightarrow k = 2$

10) $\sin\alpha = \dfrac{\sqrt{2}}{10}$



$\sqrt{98} = 7\sqrt{2}$

ربع ۲ $\Rightarrow \cos\alpha = \dfrac{-7\sqrt{2}}{10}$

$\cos\left(\dfrac{11\pi}{4}+\alpha\right) = \cos\left(\dfrac{3\pi}{4}+\alpha\right) = \cos\dfrac{3\pi}{4}\cos\alpha - \sin\dfrac{3\pi}{4}\sin\alpha = \left(-\dfrac{\sqrt{2}}{2} \times \dfrac{-7\sqrt{2}}{10}\right) - \left(\dfrac{\sqrt{2}}{2} \times \dfrac{\sqrt{2}}{10}\right) = 0.6$

17) $f\left(\dfrac{\pi}{24}\right) = 16\cos^2\left(\dfrac{\pi}{12}\right)\cos^2\left(\dfrac{\pi}{6}\right)\cos^2\left(\dfrac{\pi}{3}\right)\cos^2\left(\dfrac{2\pi}{3}\right)$

$\cos^2\dfrac{\pi}{12} = \dfrac{1+\cos\frac{\pi}{6}}{2} = \dfrac{2+\sqrt{3}}{4}$

$16\left(\dfrac{2+\sqrt{3}}{4}\right) \times \dfrac{3}{4} \times \dfrac{1}{4} \times \dfrac{1}{4}$

$= \dfrac{3(2+\sqrt{3})}{16}$