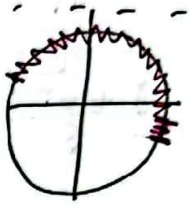
$\cot\alpha = \frac{\cos\alpha}{|\sin\alpha|}$ و $\frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$ کدام یک درست است؟ ①

① $\Rightarrow \frac{1}{|\cos\alpha|} - \frac{1-\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha} \rightarrow \frac{\sin\alpha}{|\cos\alpha|} = \frac{\sin\alpha}{\cos\alpha}$

$\Rightarrow |\cos\alpha| = \cos\alpha \Rightarrow \cos\alpha > 0$

② $\Rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \Rightarrow \sin\alpha = |\sin\alpha| \Rightarrow \sin\alpha > 0$

۲.



$\frac{-\pi}{12} < x < \frac{5\pi}{12} \Rightarrow \frac{-\pi}{6} < 2x < \frac{5\pi}{6} \Rightarrow -\frac{1}{2} < \sin 2x \leq 1$

$\Rightarrow -\frac{1}{2} < \frac{m-1}{2} \leq 1 \xrightarrow{\times 2} -2 < m-1 \leq 2 \rightarrow -1 < m \leq 3$

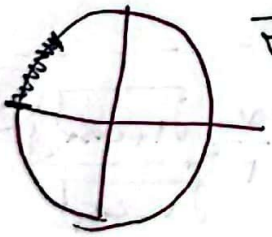
۳

$\tan x + \cot x = -2 \rightarrow \frac{1}{\sin x \cdot \cos x} = -2 \rightarrow \sin x \cdot \cos x = -\frac{1}{2}$

$(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x \cdot \cos x \rightarrow (\sin x + \cos x)^2 = 1 - \frac{2}{3} = \frac{1}{3}$

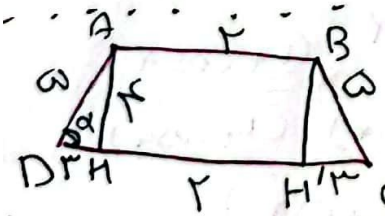
$\Rightarrow \sin x + \cos x = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ ★ $\cdot \frac{4}{(\sin x + \cos x)(\sin^2 x - \sin x \cdot \cos x + \cos^2 x)} =$

$\Rightarrow \frac{1}{\left(\frac{\sqrt{3}}{2}\right)\left(1 + \frac{1}{2}\right)} = -\frac{4}{\frac{6\sqrt{3}}{9}} = -\frac{4}{6\sqrt{3}} = \frac{9\sqrt{3}}{12}$



$\Rightarrow |\cos x| > \frac{4}{5}\sin x \Rightarrow \sin x + \cos x < 0$ ★

۴



$\cos\alpha = \frac{4}{10} \Rightarrow \frac{DH}{AD} = \frac{4}{10} \Rightarrow \frac{DH}{5} = \frac{4}{10} \Rightarrow DH = 2$

$\Rightarrow AH = 4$ ارتفاع ذوزنقه

$S = \frac{(2+8)}{2} \times 4 = 20$

$\tan\left(\frac{3\pi}{2} + \alpha\right) \cdot \tan(-\pi + \alpha) - \sin(2\pi + \alpha) \cdot \cos\left(\frac{3\pi}{2} - \alpha\right)$

$-\cot\alpha \cdot \tan\alpha - \sin\alpha \cdot \sin\alpha = -1 + \sin^2\alpha \rightarrow -1 + (1 - \cos^2\alpha)$

$\Rightarrow -\cos^2\alpha \Rightarrow k = -1$

۵ $\alpha = 10°$

$90 = 3^2 \times 10$

۶) ۲۷ ص

$$\sqrt{3}\times\cos 210^\circ\cdot\sin\left(\frac{3\pi}{2}-\alpha\right)-\sqrt{2}\times\sin 135^\circ\times\cos(\pi-\alpha)$$

$$\sqrt{3}\times-\frac{\sqrt{3}}{2}\times-\cos\alpha-\sqrt{2}\times\frac{\sqrt{2}}{2}\times-\cos\alpha=\frac{3}{2}\cos\alpha+\cos\alpha$$

$$=\frac{5}{2}\cos\alpha$$

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۷)

$$16\times\cos^2\left(\frac{\pi}{12}\right)\cdot\cos^2\left(\frac{\pi}{6}\right)\cdot\cos^2\left(\frac{\pi}{3}\right)\cdot\cos^2\left(\frac{2\pi}{3}\right)$$

$$\cos^2\frac{\pi}{12}=\frac{1+\cos\frac{\pi}{6}}{2}\Rightarrow\frac{1+\frac{\sqrt{3}}{2}}{2}=\frac{2+\sqrt{3}}{4}$$

$$16\times\frac{2+\sqrt{3}}{4}\times\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4}=\frac{6+3\sqrt{3}}{16}$$

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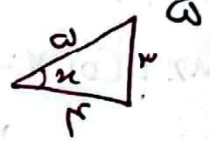
۸)

$$\frac{1-\sin x}{1+\sin x}=4\Rightarrow 1-\sin x=4+4\sin x\Rightarrow 5\sin x=-3\Rightarrow\sin x=-\frac{3}{5}$$



$$\Rightarrow\tan x\xrightarrow[\tan x>0]{\text{ربع ۳}}\tan x=\frac{3}{4}$$

$$\tan x=\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}\xrightarrow{\tan\frac{x}{2}=a}\frac{3}{4}=\frac{2a}{1-a^2}\Rightarrow 3-3a^2=8a$$

$$\Rightarrow 3a^2+8a-3=0\quad\Delta=64+36=100\quad a=\frac{-8\pm10}{6}=\frac{1}{3},-3$$

~~$\tan\frac{x}{2}=\frac{1}{3}$~~ $\quad\tan\frac{x}{2}<0$

$$\xrightarrow[\text{صحیح}]{}\tan\frac{x}{2}=-3$$

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۹)

$$\cot^2\frac{\alpha}{2}=\frac{\cos^2\frac{\alpha}{2}}{\sin^2\frac{\alpha}{2}}=\frac{\frac{1+\cos\alpha}{2}}{\frac{1-\cos\alpha}{2}}=\frac{1+\cos\alpha}{1-\cos\alpha}\Rightarrow\cot\frac{\alpha}{2}=\frac{\sqrt{1+\cos\alpha}}{\sqrt{1-\cos\alpha}}\ ★$$

$$\frac{\sin\alpha}{1-\cos\alpha}+\frac{1+\cos\alpha}{\sin\alpha}=\frac{\sin^2\alpha+1-\cos^2\alpha}{\sin\alpha(1-\cos\alpha)}=\frac{2\sin^2\alpha}{\sin\alpha(1-\cos\alpha)}\times\frac{(1+\cos\alpha)}{(1+\cos\alpha)}$$

$$\Rightarrow\frac{2\sin\alpha(1+\cos\alpha)}{1-\cos^2\alpha}=\frac{2\sin\alpha(1+\cos\alpha)}{\sin^2\alpha}=\frac{2(1+\cos\alpha)}{\sqrt{1-\cos^2\alpha}}$$

$$\frac{2\cdot\sqrt{1+\cos\alpha}\cdot\sqrt{1+\cos\alpha}}{\sqrt{1+\cos\alpha}\cdot\sqrt{1-\cos\alpha}}=\frac{2\sqrt{1+\cos\alpha}}{\sqrt{1-\cos\alpha}}\ ★=2\cot\frac{\alpha}{2}\Rightarrow\boxed{K=2}$$

$\sin\alpha = \frac{\sqrt{2}}{10}$, $\cos\alpha = 1 - \frac{2}{100} = \frac{98}{100} = \frac{\sqrt{98}}{16}$ (10)

$\frac{11\pi}{4} = \frac{8\pi}{4} + \frac{3\pi}{4} = \frac{3\pi}{2}$

$\cos\left(\frac{3\pi}{4} + \alpha\right) = \cos\frac{3\pi}{2} \cdot \cos\alpha - \sin\frac{3\pi}{2} \cdot \sin\alpha$

$= \frac{-\sqrt{2}}{2} \cdot \frac{\sqrt{98}}{10} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{10} = \frac{-\sqrt{196}}{20} - \frac{2}{20} = \frac{-14}{20}$