نام و نام خانوادگی: رزا صمدی   شماره تکلیف: ۲۸   کلاس: گروه یازدهم دختر گروه B سه‌شنبه‌ها

۱)

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|}$$

$$\frac{1}{|\cos\alpha|} - \cos\alpha = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{|\cos\alpha|} \Rightarrow \frac{\sin\alpha}{|\cos\alpha|} = \tan\alpha \to \cos\alpha > 0 \quad ①$$

$$\cot = \frac{\cos\alpha}{\sqrt{1-\cos^2\alpha}} = \frac{\cos\alpha}{\sqrt{\sin^2\alpha}} = \frac{\cos\alpha}{|\sin\alpha|} \Big\} \to \frac{\overset{>0}{\cos\alpha}}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \to \sin\alpha > 0 \quad ②$$

با توجه به ① و ② در ناحیه اول است

۲)

$$\sin 2x = \frac{m-1}{4}, \quad \frac{\pi}{12} < x < \frac{5\pi}{12} \xrightarrow{\times 2} \frac{\pi}{6} < 2x < \frac{5\pi}{6} \xrightarrow{\sin} -\frac{1}{2} < \sin 2x \le 1$$

$$\to -\frac{1}{2} < \frac{m-1}{4} \le 1 \to -2 < m-1 \le 4 \to -1 < m \le 5 \to \boxed{(-1, 5]}$$

۳)

$$① \ \tan x + \cot x = -3 \qquad 3\pi < 4x < 4\pi$$

$$① = \frac{\cos^2 x + \sin^2 x}{\sin x \cos x} = -3 \to \sin x \cos x = -\frac{1}{3}$$

$$\frac{1}{\sin^3 x + \cos^3 x} = \frac{1}{\underbrace{(\sin x + \cos x)}_{②}(\sin^2 + \cos^2 - \sin x\cos x)} = \frac{1}{\frac{-1}{\sqrt{3}} \times \frac{4}{3}} = \boxed{-\frac{3\sqrt{3}}{4}}$$

$$② \ (\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2\sin x\cos x = 1 + 2\left(-\frac{1}{3}\right) = \frac{1}{3} \to \sin x + \cos x \begin{cases} \frac{1}{\sqrt{3}} \ ✗ \\ -\frac{1}{\sqrt{3}} \ ✓ \end{cases} \Rightarrow \sin x + \cos x = \frac{-1}{\sqrt{3}}$$

$$③ \ 3\pi < 4x < 4\pi \xrightarrow{\div 4} \frac{3\pi}{4} < x < \pi \to |\sin x| < |\cos x|$$

۴)

$$\cos\theta = \frac{6}{10} \to \frac{DH}{AD} = \frac{6}{10} \to DH = 3 = PC$$

فیثاغورس $\Rightarrow AH = \sqrt{25+9} = 4$

$$S = \frac{(2+8)\times 4}{2} = \boxed{20}$$

۵)

$$\tan(285)\tan(565) - \sin(1095)\cos(255)$$

$$\hookrightarrow \tan(270+15)\tan(-180+15) - \sin(1080+15)\cos(270-15)$$

$$\hookrightarrow (-\cot 15)(\tan 15) - (\sin 15)(-\sin 15) = -1 + \sin^2 15 = -1(1-\sin^2 15)$$

$$= -\cos^2 15 = k\cos^2 15 \to \boxed{k = -1}$$

$A = \sqrt{۳}\cos(۲۱۰)\sin(۲۴۳) - \sqrt{۲}\sin(۱۳۵)\cos(۱۵۳) =$

$A = \sqrt{۳}\,(\cos(۱۸۰+۳۰))\sin(۲۷۰-۲۷) - \sqrt{۲}\,(\sin(۹۰+۴۵))\cos(۱۸۰-۲۷) =$

$= \sqrt{۳}(-\cos۳۰)(-\cos۲۷) - \sqrt{۲}(\sin۴۵)(-\cos۲۷) = \sqrt{۳}\left(\frac{\sqrt{۳}}{۲}\right)(\cos۲۷) + \sqrt{۲}\times\frac{\sqrt{۲}}{۲}(\cos۲۷)$

$= \frac{۵}{۲}(\cos۲۷) \Rightarrow \frac{\frac{۵}{۲}\cos۲۷}{\cos۲۷} = \frac{۵}{۲}$ (۶

$f(x) = ۱۶\cos^۲(۲x)\cos^۲(۴x)\cos^۲(۸x)\cos^۲(۱۶x)$ $\quad F\left(\frac{\pi}{۲۴}\right)$

$\hookrightarrow F\left(\frac{\pi}{۲۴}\right) = ۱۶\cos^۲\left(\frac{\pi}{۱۲}\right)\cos^۲\left(\frac{\pi}{۶}\right)\cos^۲\left(\frac{\pi}{۳}\right)\cos^۲\left(\frac{۲\pi}{۳}\right)$

$\hookrightarrow ۱۶\times\frac{\sqrt{۳}+۲}{۴}\times\frac{۳}{۴}\times\frac{۱}{۴}\times-\frac{۱}{۴} = \frac{-۶+۳\sqrt{۳}}{۱۶}$ (۷

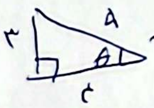
$\cos^۲\left(\frac{\pi}{۱۲}\right) = \cos^۲۱۵ = \frac{۱+\cos۳۰}{۲}$

$\hookrightarrow \cos^۲۱۵ = \left(\frac{\sqrt{۲+\sqrt{۳}}}{۲}\right)^۲ = \frac{۲+\sqrt{۳}}{۴}$

$\frac{۱-\sin x}{۱+\sin x} = ۴$ $\quad \tan\frac{x}{۲} = ? \rightarrow \tan\frac{x}{۲} = \frac{\sin x}{۱+\cos x} = \frac{-\frac{۳}{۵}}{۱-\frac{۴}{۵}} = -۳$

$\hookrightarrow ۱-\sin x = ۴+۴\sin x \rightarrow -۳ = ۵\sin x \rightarrow \sin x = \frac{-۳}{۵}$

(۸



$\rightarrow \cos x = \frac{۴}{۵} \xrightarrow[\text{سوم}]{\text{ربع}} -\frac{۴}{۵}$

$\frac{\sin\theta}{۱-\cos\theta} + \frac{۱+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{۲}$

① $\frac{\sin\theta}{۱+\cos\theta} = \tan\frac{\theta}{۲} \xrightarrow{\text{معکوس}} \frac{۱+\cos\theta}{\sin\theta} = \cot\frac{\theta}{۲}$

② $\frac{۱-\cos\theta}{\sin\theta} = \tan\frac{\theta}{۲} \xrightarrow{\text{معکوس}} \frac{\sin\theta}{۱-\cos\theta} = \cot\frac{\theta}{۲}$

①② $\Rightarrow \cot\frac{\theta}{۲} + \cot\frac{\theta}{۲} = ۲\cot\frac{\theta}{۲}$ (۹

$\boxed{k = ۲}$

$\cos\left(\frac{۱۱\pi}{۴}+\alpha\right)$ $\quad \cos^۲\alpha + \sin^۲\alpha = ۱ \xrightarrow{\text{ربع دوم}} \cos\alpha = -\sqrt{۱-\sin^۲\alpha} = -\sqrt{\frac{۴۹}{۵۰}} = \frac{-۷}{۵\sqrt{۲}}$

$\hookrightarrow \cos\left(۲\pi + \frac{۳\pi}{۴} + \alpha\right) = \cos\left(\frac{۳\pi}{۴}+\alpha\right) = \cos\alpha\cos\frac{۳\pi}{۴} - \sin\alpha\sin\frac{۳\pi}{۴}$

$\Rightarrow \left(\frac{-۷}{۵\sqrt{۲}}\times\frac{\sqrt{۲}}{۲}\right) - \left(\frac{\sqrt{۲}}{۱۰}\times\frac{\sqrt{۲}}{۲}\right) = \frac{۷}{۱۰} - \frac{۱}{۱۰} = \frac{۶}{۱۰} = \frac{۳}{۵}$ (۱۰