۲۰

(۱)

$$\frac{1}{\sqrt{\cos^2\alpha}} - \frac{1}{\cot\alpha} = \frac{1}{|\cos\alpha|} - \frac{\sin\alpha}{\cos\alpha} = \frac{1-\sin\alpha}{|\cos\alpha|} \rightarrow \frac{-\sin\alpha}{\cos\alpha} = \frac{-\sin\alpha}{|\cos\alpha|}$$

$\rightarrow \cos\alpha > 0 \quad I$

$$\cot\alpha = \frac{\cos\alpha}{\underbrace{\sqrt{1-\cos^2\alpha}}_{\sin^2\alpha}} \rightarrow \frac{\cos\alpha}{\sin\alpha} = \frac{\cos\alpha}{|\sin\alpha|} \rightarrow \sin\alpha > 0 \; II$$

$I \cap II \rightarrow$ ۱ ربع

۵

(۲)

$$\frac{-\pi}{12} < x < \frac{5\pi}{12} \rightarrow -\frac{\pi}{6} < 2x < \frac{5\pi}{6} \rightarrow -\frac{1}{2} < \underbrace{\sin^2 x}_{\frac{m-1}{4}} \leq 1$$

$$\rightarrow -\frac{1}{4} < \frac{m}{4} \leq \frac{5}{4} \quad \boxed{-1 < m \leq 5}$$

۵

(۳)

$$\tan x + \cot x = \frac{1}{\sin x \cos x} = -3 \quad , \quad (\sin x + \cos x)^2 = 1 - \frac{2}{3} = \frac{1}{3}$$

$$(\sin x - \cos x)^2 = 1 + \frac{2}{3} = \frac{5}{3}$$

$$\frac{3\pi}{4} < x < \pi \qquad \frac{1}{(\sin + \cos)(\sin^2 - \sin\cos + \cos^2)} = \boxed{-\frac{3\sqrt{3}}{4}}$$

$\sin + \cos = -\frac{1}{\sqrt{3}}$, $1 - (-\frac{1}{3}) = \frac{4}{3}$

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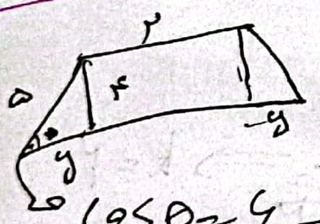
(۴)



$\cos\theta = \frac{y}{5} \rightarrow y = 3$ قاعده بزرگ $= 2y + 2 = 8$ مساحت ذوزنقه $= \frac{(8+2)\times 4}{2} = 20$

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(۵)

$$\tan(270 + 15) \times \tan(-180 + 15) - \sin 15 \times \cos(270 - 15)$$

$$= -\cot 15 \times \tan 15 + \sin^2 15 \rightarrow -1 + \underbrace{\sin^2 15}_{1-\cos^2 15} = -\cos^2 15$$

$$\sim \boxed{K = -1}$$

۵

(٦)

$-\sin ٦٣$

$\sqrt{٣}\cos ٢١٠ \times \sin ٢٤٣ - \sqrt{٢}\sin(١٣٥)\cos(١٥٣)$

$\Rightarrow -\sqrt{٣} \times \frac{\sqrt{٣}}{٢} \times \cos ٢٧ - \sqrt{٢} \times \frac{\sqrt{٢}}{٢} \times -\cos ٢٧$ (٤)

$-\frac{٣}{٢}$ $\quad +١$

$= -\frac{٣}{٢}\cos ٢٧ + \cos ٢٧ = \cos ٢٧(-\frac{١}{٢}) \to -\frac{١}{٢} \times \frac{\cos ٢٧}{\cos ٢٧}$ $\boxed{\cos ٢٧ = ?}$

(٧)

$f(\frac{\pi}{٢٤}) = ١٦\cos^٢(\frac{\pi}{١٢})\cos^٢(\frac{\pi}{٦})\cos^٢(\frac{\pi}{٣})\cos^٢(٢\frac{\pi}{٣})$

$* \cos^٢ ١٥ = \frac{١+\cos ٣٠}{٢} = \frac{٢+\sqrt{٣}}{٤}$ (٢)

$١٦(\frac{٢+\sqrt{٣}}{٤}) \times \frac{٣}{٤} \times \frac{١}{٤} \times \frac{١}{٤} = \frac{٣}{٢٨}(٢+\sqrt{٣}) = \frac{٦+٣\sqrt{٣}}{٢٨} = \frac{٦+٣\sqrt{٣}}{١٦}$

(٨)

طرفین وسطین $\to ١ - \sin x = ٤ + ٤\sin x \to \sin x = -\frac{٣}{٥}$ $\xrightarrow{*\ \sin^٢+\cos^٢=١}$

$\cos x = -\frac{٤}{٥}$ ، $\sin ٢\theta = \frac{٢\tan\theta}{١+\tan^٢\theta} \Rightarrow \sin x = \frac{٢\tan\frac{x}{٢}}{١+\tan^٢\frac{x}{٢}} = -\frac{٣}{٥}$ *در ربع سوم $\cos$ و $\sin<٠$

$١٠\tan\frac{x}{٢} = -٣ - ٣\tan^٢\frac{x}{٢} \to ٣\tan^٢\frac{x}{٢} + ١٠\tan\frac{x}{٢} + ٣ = ٠$ (٣)

$(٣\tan\frac{x}{٢}+١)(\tan\frac{x}{٢}+٣)=٠ \to \tan\frac{x}{٢} = -\frac{١}{٣}$ ✗

$\quad \to \tan\frac{x}{٢} = \boxed{-٣}$ ✓

(٩)

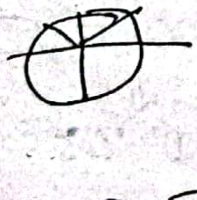
$\frac{\sin\theta}{١-\cos\theta} + \frac{١+\cos\theta}{\sin\theta} = k\cot\frac{\theta}{٢}$

$\downarrow$

$\frac{١}{\tan\frac{\theta}{٢}} + \frac{١}{\tan\frac{\theta}{٢}} = \frac{٢}{\tan\frac{\theta}{٢}} = \frac{k}{\tan\frac{\theta}{٢}} \Rightarrow k=٢$ (٤)

(١٠)

$\cos(١١\frac{\pi}{٤}+\alpha) = \cos(\frac{٣}{٤}\pi + \alpha)$ (٤)



$= \cos\frac{٣}{٤}\pi \times \cos\alpha - \sin\frac{٣}{٤}\pi \times \sin\alpha$

$= -\frac{\sqrt{٢}}{٢} \times -\frac{٧\sqrt{٢}}{١٠} - \frac{\sqrt{٢}}{٢} \times \frac{\sqrt{٢}}{١٠} = ٠/٦$