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نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۳ ..... کلاس .....

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$\lim_{x\to 1} \frac{4x^2-7x+3}{5x^2-8x+3} = \frac{0}{0}$ رفع ابهام $\Rightarrow \frac{(x-1)(4x-3)}{(x-1)(5x-3)} = \frac{4x-3}{5x-3}$

$x\to 1 \Rightarrow \frac{4-3}{5-3} = \boxed{\frac{1}{2}}$

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$\lim_{x\to 0} \frac{|2x-1|-|2x+1|}{x}$ $\quad \frac{1-2x-2x-1}{x} = \frac{-4x}{x} = -4$

$|\quad \Rightarrow 2x-1<0$

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$\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0}$ رفع ابهام $\Rightarrow \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}-2)} = \sqrt{x}+2 \xrightarrow{x\to 4} 2+2 = 4$

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$\lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6}$ $\quad \frac{\sqrt{x}(\sqrt{x}-\sqrt{2})}{(2x+3)(x-2)} = \frac{\sqrt{x}(\sqrt{x}-\sqrt{2})}{(2x+3)(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})} \Rightarrow \frac{\sqrt{2}}{7\times 4\sqrt{2}} = \frac{1}{14}$

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$\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} = \frac{0}{0}$ رفع ابهام $\Rightarrow \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}} = \frac{1-x}{4-5+x} \times \frac{4}{2}$

$x-1 \to -(1-x)$

$= -\frac{4}{2} = -2$

یا تشریحی تر

$\frac{1-x}{4-5+x} \times \frac{2+\sqrt{5-x}}{1+\sqrt{x}} \xrightarrow{x=1} -1 \times \frac{2+2}{2} = -2$

$\to -(1-x)$

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$$\lim_{x\to 4}\frac{\sqrt{x+4}-\sqrt{8-x}}{\sqrt[3]{2x+1}-3}=\frac{0}{0}$$ مزدوج

$$\frac{\sqrt{x+4}-\sqrt{8-x}}{\sqrt[3]{2x+1}-3}\times\frac{\sqrt{x+4}+\sqrt{8-x}}{\sqrt{x+4}+\sqrt{8-x}}\times\frac{\sqrt[3]{(2x+1)^2}+9+3\sqrt[3]{2x+1}}{\sqrt[3]{(2x+1)^2}+9+3\sqrt[3]{2x+1}}$$

$x+4-8+x \to 2(x-4)$

$$=\frac{2x+4-17}{2x+1-27}\times\frac{27}{8}=\frac{2(x-4)}{2(x-13)}\times\frac{27}{8}=\frac{81}{40}$$

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$$\lim_{x\to 1}\frac{\sqrt{x+\sqrt{x}}-2}{\sqrt[3]{x}-1}=\frac{0}{0}$$ مزدوج

$$\frac{\sqrt{x+\sqrt{x}}-2}{\sqrt[3]{x}-1}\times\frac{\sqrt{x+\sqrt{x}}+2}{\sqrt{x+\sqrt{x}}+2}\times\frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}}$$

$$\Rightarrow\frac{x+\sqrt{x}-4}{x-1}\times\frac{3}{4}=\frac{(\sqrt{x})^2+\sqrt{x}-4}{x-1}\times\frac{3}{4}=\frac{(3\sqrt{x}+4)(\sqrt{x}-1)}{x-1}\times\frac{3}{4}=\frac{(3\sqrt{x}+4)(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)}\times\frac{3}{4}$$

$x=1 \Rightarrow \frac{3+4}{1+1}\times\frac{3}{4}=\frac{21}{8}$

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$$\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}=\frac{0}{0}$$ تجزیه

$$\frac{(1+\cos x)(1+\cos^2 x-\cos x)}{1-\cos^2 x}=\frac{(1+\cos x)(1+\cos^2 x-\cos x)}{(1-\cos x)(1+\cos x)}$$

$x\to\pi \Rightarrow \frac{1+(-1)^2-(-1)}{1-(-1)}=\frac{3}{2}$

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$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{0}{0}$$ مزدوج

$$\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}=\frac{\frac{\cos x-\sin x}{\cos x}}{\sin x-\cos x}=\frac{-(\sin x-\cos x)}{\cos x(\sin x-\cos x)}$$

$$=\frac{-1}{\cos x}\xrightarrow{x\to\frac{\pi}{4}}\frac{-1}{\frac{\sqrt{2}}{2}}=\frac{-2}{\sqrt{2}}=-\sqrt{2}$$

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$$\lim_{x\to\frac{3\pi}{4}}\frac{\tan^2 x-1}{\cos^2 x}$$

$\tan^2\alpha=\frac{1-\cos^2\alpha}{1+\cos^2\alpha}$

$-2\cos^2 x$

$$\frac{\frac{1-\cos^2 x}{1+\cos^2 x}-1}{\cos^2 x}=\frac{\frac{1-\cos^2 x-1-\cos^2 x}{1+\cos^2 x}}{\cos^2 x}=\frac{-2}{1+\cos^2 x}\xrightarrow{x\to\frac{3\pi}{4}}\frac{-2}{1+\cos^2\frac{3\pi}{4}}=-2$$