نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..... کلاس .....

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$$\lim_{x\to 1}\frac{4x^2-7x+3}{5x^2-1x+3}=\frac{0}{0}\xrightarrow{\text{تجزیه}}\frac{(x-1)(4x-3)}{(x-1)(5x-3)}=\frac{4x-3}{5x-3}$$

$$\xrightarrow{x\to 1}\frac{4-3}{5-3}=\boxed{\frac{1}{2}}$$

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$$\lim_{x\to 0}\frac{|2x-1|-|2x+1|}{x}\qquad \frac{1-2x-2x-1}{x}=\frac{-4x}{x}=-4$$

$|2x-1|<0 \Leftarrow$

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$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\frac{0}{0}\xrightarrow{\text{تجزیه}}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}-2)}=\sqrt{x}+2\xrightarrow{x\to 4}2+2=4$$

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$$\lim_{x\to 2}\frac{x-\sqrt{2x}}{2x^2-x-6}\qquad \frac{\sqrt{x}(\sqrt{x}-\sqrt{2})}{(2x+3)(x-2)}=\frac{\sqrt{x}(\sqrt{x}-\sqrt{2})}{(2x+3)(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})}\Rightarrow\frac{\sqrt{2}}{7\times 2\sqrt{2}}=\frac{1}{14}$$

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$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}=\frac{0}{0}\xrightarrow{\text{تجزیه}}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}\times\frac{1+\sqrt{x}}{1+\sqrt{x}}\times\frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}=\frac{1-x}{4-5+x}\times\frac{4}{2}$$

$x-1\hookrightarrow -(1-x)$

$$=-\frac{4}{2}=-2$$

یا تشریحی تر

$$\frac{1-x}{4-5+x}\hookrightarrow -(1-x)\qquad \frac{1-x}{1+\sqrt{x}}\times\frac{2+\sqrt{5-x}}{x=1}\Rightarrow -1\times\frac{2+2}{2}=-2$$

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$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}=\frac{0}{0}$$ مزدوج

$$\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}\times\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\times\frac{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}{\sqrt[3]{(5x+7)^2}+9+3\sqrt[3]{5x+7}}$$

$3x+4\to 3(x-4)$

$5x-20\to 5(x-4)$

$$=\frac{3x+4-16}{5x+7-27}\times\frac{27}{8}=\frac{3(x-4)}{5(x-4)}\times\frac{27}{8}=\frac{81}{40}$$

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$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}=\frac{0}{0}$$ مزدوج

$$\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}\times\frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2}\times\frac{\sqrt[3]{x^2}+1+\sqrt[3]{x}}{\sqrt[3]{x^2}+1+\sqrt[3]{x}}$$

$$\Rightarrow\frac{3x+\sqrt{x}-4}{x-1}\times\frac{3}{4}=\frac{3(\sqrt{x})^2+\sqrt{x}-4}{x-1}\times\frac{3}{4}=\frac{(3\sqrt{x}+4)(\sqrt{x}-1)}{x-1}\times\frac{3}{4}=\frac{(3\sqrt{x}+4)(\sqrt{x}-1)}{(\sqrt{x}-1)(\sqrt{x}+1)}\times\frac{3}{4}$$

$x=1\Rightarrow\frac{3+4}{1+1}\times\frac{3}{4}=\frac{21}{8}$

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$$\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}=\frac{0}{0}$$ مزدوج

$$\frac{(1+\cos x)(1+\cos^2 x-\cos x)}{1-\cos^2 x}=\frac{(1+\cos x)(1+\cos^2 x-\cos x)}{(1-\cos x)(1+\cos x)}$$

$$x\to\pi\Rightarrow\frac{1+(-1)^2-(-1)}{1-(-1)}=\frac{3}{2}$$

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$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{0}{0}$$ مزدوج

$$\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}=\frac{\frac{\cos x-\sin x}{\cos x}}{\sin x-\cos x}=\frac{-(\sin x-\cos x)}{\cos x(\sin x-\cos x)}$$

$$=\frac{-1}{\cos x}\xrightarrow{x\to\frac{\pi}{4}}\frac{-1}{\frac{\sqrt{2}}{2}}=\frac{-2}{\sqrt{2}}=-\sqrt{2}$$

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$$\lim_{x\to\frac{3\pi}{4}}\frac{\tan^2 x-1}{\cos^2 x}$$

$$\tan^2\alpha=\frac{1-\cos 2\alpha}{1+\cos 2\alpha}$$

$$\frac{\frac{1-\cos 2x}{1+\cos 2x}-1}{\cos^2 x}=\frac{\frac{1-\cos 2x-1-\cos 2x}{1+\cos 2x}}{\cos^2 x}=\frac{-2}{1+\cos 2x}\xrightarrow{x\to\frac{3\pi}{4}}\frac{-2}{1+\cos\frac{3\pi}{2}}=-2$$