

صحيح ضرب بسط ومقام

$$\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 - 1x + 3} = \frac{x(x - \frac{3}{2})(2x-1)}{x(x - \frac{3}{2})(2x-1)} = \frac{1}{\frac{1}{1 - \frac{3}{2}}} = \frac{1}{\frac{2}{1-3}} = \frac{1}{-\frac{2}{2}} = -\frac{1}{2}$$

ملاحظة: لا بد من ضرب بسط ومقام

$$\lim_{n \rightarrow \infty} \frac{|3n-1| - |3n+1|}{n} = \frac{3n-1 - 3n-1}{n} = \frac{-2}{n} = 0$$

تطبيق قاعدة ل'Hopital

$$\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} \times \frac{\sqrt{x}+2}{\sqrt{x}+2} = \frac{(x-4)(\sqrt{x}+2)}{x-4} = \sqrt{x}+2 = 4$$

$$\lim_{x \rightarrow 2} \frac{x - \sqrt{x}}{x^2 - x - 2} \times \frac{x + \sqrt{x}}{x + \sqrt{x}} = \frac{x^2 - x}{(x-2)(x+2)} \times \frac{x + \sqrt{x}}{x + \sqrt{x}} = \frac{x}{x+2} = \frac{2}{4} = \frac{1}{2}$$

$$\lim_{x \rightarrow 16} \frac{1 - \sqrt{x}}{4x - \sqrt{4x-2}} \times \frac{1 + \sqrt{x}}{1 + \sqrt{x}} \times \frac{4x + \sqrt{4x-2}}{4x + \sqrt{4x-2}} = \frac{(1-x)(1+\sqrt{x})(4x+\sqrt{4x-2})}{(4x-\sqrt{4x-2})(1+\sqrt{x})(4x+\sqrt{4x-2})} = \frac{1-x}{4x-\sqrt{4x-2}}$$

$$\lim_{x \rightarrow 4} \frac{\sqrt{4x+5} - 4}{\sqrt{4x+5} - 3} \times \frac{\sqrt{4x+5} + 4}{\sqrt{4x+5} + 4} \times \frac{\sqrt{4x+5} + 3}{\sqrt{4x+5} + 3} = \frac{(\sqrt{4x+5} - 4)(\sqrt{4x+5} + 4)(\sqrt{4x+5} + 3)}{(\sqrt{4x+5} - 3)(\sqrt{4x+5} + 4)(\sqrt{4x+5} + 3)} = \frac{16-16}{(4-3)(4+3)(4+3)} = \frac{0}{(1)(7)(7)} = 0$$

$$\lim_{n \rightarrow 1} \frac{\sqrt{3n+\sqrt{n}} - 2}{\sqrt{n} - 1} \times \frac{\sqrt{3n+\sqrt{n}} + 2}{\sqrt{3n+\sqrt{n}} + 2} \times \frac{\sqrt{n} + 1}{\sqrt{n} + 1} = \frac{(\sqrt{3n+\sqrt{n}} - 2)(\sqrt{n} + 1)}{(\sqrt{n} - 1)(\sqrt{3n+\sqrt{n}} + 2)(\sqrt{n} + 1)} = \frac{(\sqrt{3n+\sqrt{n}} - 2)(\sqrt{n} + 1)}{(\sqrt{n} - 1)(\sqrt{3n+\sqrt{n}} + 2)(\sqrt{n} + 1)}$$

$$= \frac{1}{\sqrt{n}} \times \frac{3n + \sqrt{n} - 4}{n-1} \times \frac{(\sqrt{3n+\sqrt{n}} - 2)(\sqrt{n} + 1)}{(\sqrt{3n+\sqrt{n}} + 2)(\sqrt{n} + 1)} = \frac{1}{\sqrt{n}} \times \frac{3n + \sqrt{n} - 4}{n-1} \times \frac{(\sqrt{3n+\sqrt{n}} - 2)(\sqrt{n} + 1)}{(\sqrt{3n+\sqrt{n}} + 2)(\sqrt{n} + 1)}$$

$$= \frac{1}{\sqrt{n}} \times \frac{9n^2 + 19 - 2\sqrt{3n} - 4n}{(n-1)} = \frac{1}{\sqrt{n}} \times \frac{5n^2 + 19 - 2\sqrt{3n}}{(n-1)}$$

$$\lim_{n \rightarrow \pi} \frac{1 + \cos n}{\sin n} = \frac{1 + \cos n}{1 - \cos n} = \frac{(1 + \cos n)(1 + \cos n)}{(1 - \cos n)(1 + \cos n)} = \frac{1 + \cos n}{1 - \cos n}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \tan x}{\sin x - \cos x} \times \frac{\cos x}{\cos x} = \frac{\cos x - \sin x - 1}{\cos x (\sin x - \cos x)} = -\sqrt{2} \quad (9)$$

$$\lim_{x \rightarrow \frac{3\pi}{4}} \frac{\tan x - 1}{\cos(x)} \times \frac{\cos x}{\cos x} = \frac{\sin x - \cos x}{\cos x (\cos x - \sin x)} = -1 \quad (10)$$