① $\lim_{x\to 1} \frac{4x^2-7x+3}{5x^2-1x+3} = \frac{(x-1)(4x-3)}{(x-1)(5x-3)}$

$\frac{4-3}{5-3} = \frac{1}{2}$

② $\lim_{x\to 0} \frac{|3x-1|-|3x+1|}{x} =$

$\frac{1-3x-3x-1}{x} = \frac{-6x}{x} = -6$

③ $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = 4$

④ $\lim_{x\to 2} \frac{x-\sqrt{3x}}{2x^2-x-6} \times \frac{x+\sqrt{3x}}{x+\sqrt{3x}} = \frac{x^2-3x}{(x-2)(2x+3)(x+\sqrt{3x})}$

$\frac{x(x-3)}{(x-2)(2x+3)(x+\sqrt{3x})} = \frac{2}{7\times 4} = \frac{1}{14}$

⑤ $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}$

$= \frac{1-x}{4-5+x} \times \frac{2+\sqrt{5-x}}{1+\sqrt{x}} = \frac{-4}{2} = -2$

(4)

$$\lim_{x \to 4} \frac{\sqrt{2x+4} - 4}{\sqrt[3]{8x+19} - 3} \quad \text{حالة}$$

$$\frac{2\sqrt{(2x+4)}}{3\sqrt[3]{(8x+19)^2}}$$

$$= \frac{2 \times 2 \times 19}{2\sqrt{2 \times 4 + 4} \; 3\sqrt[3]{(8 \times 4 + 19)^2}}$$

$$= \frac{19}{40}$$

(٧)

$$\lim_{x \to 1} \frac{\sqrt{x^2 + \sqrt{x}} - \sqrt{2}}{x - 1}$$

$$= \frac{\frac{1}{2\sqrt{2}} + 2}{1}$$

$$= \frac{\sqrt{x^2 + \sqrt{x}} + \sqrt{2}}{\sqrt{x^2 + \sqrt{x}} + \sqrt{2}} \times \frac{\sqrt{x^2 + \sqrt{x}} - \sqrt{2}}{x - 1}$$

$$= \frac{x^2 + \sqrt{x} - 2}{(x-1)\left(\sqrt{x^2 + \sqrt{x}} + \sqrt{2}\right)}$$

$$= \frac{(x-1)(x+2)}{(x-1)\left(\sqrt{x^2 + \sqrt{x}} + \sqrt{2}\right)}$$

$$= \frac{x+2}{\sqrt{x^2 + \sqrt{x}} + \sqrt{2}}$$

$$= \frac{3}{2\sqrt{2}}$$

$$\therefore \lim_{x \to 1} \frac{\sqrt{x^2 + \sqrt{x}} - \sqrt{2}}{x - 1} = \frac{3}{2\sqrt{2}}$$

(1)

$$\lim_{x \to \pi} \frac{1+\cos^2 x}{\sin^2 x} = \frac{(1+\cos x)(1+\cos^2 x - \cos x)}{1-\cos^2 x}$$

$$= \frac{(1+\cos x)(1+\cos^2 x - \cos x)}{(1-\cos x)(1+\cos x)}$$

$$\frac{1+\cos^2 \pi - \cos \pi}{1-\cos \pi} = \frac{3}{2}$$

(9

$$\lim_{x \to \frac{\pi}{4}} \frac{1-\tan x}{\sin x - \cos x} = \frac{-(1\alpha(1+\tan^2 x)\alpha 1)}{1\alpha\cos x\alpha 1 - (1\alpha(-\sin x)\alpha 1)}$$

$$\frac{-2}{2\alpha\frac{\sqrt{2}}{2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2}$$

$$\lim_{x \to \frac{\pi}{4}} \frac{\tan^2 x - 1}{\cos 2x} = \frac{(\tan x - 1)(\tan x + 1)}{(\cos x - \sin x)(\cos x + \sin x)}$$

(10)

$$- \frac{(\sin x + \cos x)(\cos x - \sin x)}{\cos 2x (\cos x - \sin x)(\cos x + \sin x)} = \frac{-1}{\cos^2 x} = -2$$