نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۳۰ ..... کلاس .....

۱

$$\lim_{x\to 1}\frac{4x^2-7x+3}{5x^2-8x+3}=\frac{0}{0}\to\frac{(x-1)(4x-3)}{(x-1)(5x-3)}=\frac{1}{2}$$

۲

$$\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}=\frac{0}{0}\to\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}=\frac{|<3x-3x-1}{x}=\frac{-4x}{x}=-4$$

۳

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\frac{0}{0}\to\lim\frac{x-4}{\sqrt{x}-2}\times\frac{\sqrt{x}+2}{\sqrt{x}+2}=\frac{(x-4)(\sqrt{x}+2)}{x-4}=\sqrt{4}+2=4$$

۴

$$\lim_{x\to 2}\frac{x-\sqrt{2x}}{2x^2-x-6}=\frac{0}{0}\to\lim_{x\to 2}\frac{x-\sqrt{2x}}{(x-2)(2x+3)}\times\frac{x+\sqrt{2x}}{x+\sqrt{2x}}=\frac{x^2-2x}{(x-2)(2x+3)(x+\sqrt{2x})}=\frac{x(x-2)}{(x-2)(2x+3)(x+\sqrt{2x})}$$

$$\frac{2}{7\times 4}=\frac{1}{14}$$

۵

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}=\frac{0}{0}\to\lim_{x\to 1}\frac{1-\sqrt{x}}{2-\sqrt{5-x}}\times\frac{1+\sqrt{x}}{1+\sqrt{x}}\times\frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}=$$

$$-1\times\frac{1-x}{4-5+x}\times\frac{2+\sqrt{5-x}}{1+\sqrt{x}}=-1\times\frac{2+\sqrt{5-1}}{1+\sqrt{1}}=-1\times\frac{4}{2}=-2$$

٦

$$\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}=\frac{0}{0}\to\lim_{x\to 4}\frac{\sqrt{3x+4}-4}{\sqrt[3]{5x+7}-3}\times\frac{\sqrt{3x+4}+4}{\sqrt{3x+4}+4}\times\frac{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}$$

$$3x-12=3(x-4)$$

$$=\frac{3x+4-16}{5x+7-27}\times\frac{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}{\sqrt{3x+4}+4}=\frac{3}{5}\times\frac{9+9+9}{8}=\boxed{\frac{27}{40}}$$

$5x-20$, $5(x-4)$

٧

$$\lim_{x\to 1}\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}=\frac{0}{0}\to\frac{\sqrt{3x+\sqrt{x}}-2}{\sqrt[3]{x}-1}\times\frac{\sqrt{3x+\sqrt{x}}+2}{\sqrt{3x+\sqrt{x}}+2}\times\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$$

$$=\frac{3x+\sqrt{x}-4}{x-1}\times\frac{3}{4}=\frac{(\sqrt{x}-1)(3\sqrt{x}+4)}{(\sqrt{x}+1)(\sqrt{x}-1)}=\frac{7}{2}\times\frac{3}{4}=\boxed{\frac{21}{8}}$$

٨

$$\lim_{x\to\pi}\frac{1+\cos^3 x}{\sin^2 x}=\lim_{x\to\pi}\frac{(1+\cos x)(1+\cos^2 x-\cos x)}{1-\cos^2 x}=\frac{1+\cos^2 x-\cos x}{1-\cos x}=$$

$$(1+\cos x)(1-\cos x)$$

$$\frac{1+1-(-1)}{1-(-1)}=\boxed{\frac{3}{2}}$$

٩

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{\sin x-\cos x}=\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}=\frac{\frac{\cos x-\sin x}{\cos x}}{\sin x-\cos x}=\frac{-1}{\cos x}=\frac{-1}{\frac{\sqrt{2}}{2}}=\frac{-2\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}=$$

$$\frac{-2\sqrt{2}}{2}=\boxed{-\sqrt{2}}$$

١٠

$$\lim_{x\to\frac{\pi}{4}}\frac{\tan^2 x-1}{\cos 2x}=\frac{0}{0}\to\lim\frac{(\tan x+1)(\tan x-1)}{\cos^2 x-\sin^2 x}=\frac{\left(\frac{\sin x}{\cos x}+1\right)\left(\frac{\sin x}{\cos x}-1\right)}{\cos^2 x-\sin^2 x}=\frac{\left(\frac{\sin x+\cos x}{\cos x}\right)\left(\frac{\sin x-\cos x}{\cos x}\right)}{\cos^2 x-\sin^2 x}$$

$$=\lim_{x\to\frac{\pi}{4}}\frac{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}{\cos^2 x-\sin^2 x}=\frac{-1}{\cos^2 x}=\frac{-1}{\left(\frac{\sqrt{2}}{2}\right)^2}=\boxed{-2}$$