Year: Month: Date: ( ) Subject: سانیا جاوری

۱۰

(۱) $\lim_{x\to 1} \frac{۴x^۲ - ۷x + ۳}{۵x^۲ - ۸x + ۳}$

$۵A = (۵x)^۲ - ۸(۵x) + ۱۵ = (۵x - ۳)(۵x - ۵)$

$A = ۴x^۲ - ۷x + ۳$

$۴A = (۴x)^۲ - ۷ \times ۴x + ۱۲ = (۴x - ۳)(۴x - ۴)$

$\lim_{x\to 1} \frac{(۴x - ۳)(x - ۱)}{(۵x - ۳)(x - ۱)} = \frac{۱}{۲}$ ۹

(۲) $\lim_{x\to ۰^+} \frac{|۳x - ۱| - |۳x + ۱|}{x} = \frac{-۳x + ۱ - ۳x - ۱}{x}$ ۹

$\lim_{x\to ۰^-} \frac{|۳x - ۱| - |۳x + ۱|}{x} = \frac{۳x + ۱ - ۳x - ۱}{x}$

-۶

(۳) $\lim_{x\to ۴} \frac{(\sqrt{x} - ۲)(\sqrt{x} + ۲)}{\sqrt{x} - ۲} = \lim_{x\to ۴} \sqrt{x} + ۲ = ۴$ ۹

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(۴) $\lim_{x\to 2} \frac{x-\sqrt{2x}}{2x^2-x-6} \times \frac{x+\sqrt{2x}}{x+\sqrt{2x}} = \frac{x-2x}{(2x^2-x-6)(x+\sqrt{2x})} = \frac{x}{(2x+3)(x-2)(x+\sqrt{2x})}$

$x \to 2$

$\frac{2}{28} = \frac{1}{14}$

$A = 2x^2 - x - 6$

$2A = 4x^2 - 2x - 12 = (2x+3)(2x-4)$

$2A = 2(2x+3)(x-2)$

(۵) $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{3-x}} = \frac{0}{0}$ ss $\frac{1-\sqrt{x}}{2-\sqrt{3-x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} = \frac{1-x}{(2-\sqrt{3-x})(1+\sqrt{x})} \times \frac{2+\sqrt{3-x}}{2+\sqrt{3-x}}$

$1-\sqrt{1} = 0$

$2-\sqrt{3-1} = 2-\sqrt{2} \neq 0$

$-\frac{1-x}{(x-1)(1+\sqrt{x})}(2+\sqrt{3-x})$

$\frac{-1}{1+\sqrt{x}}(2+\sqrt{3-x}) = -\frac{1}{2}(2+\sqrt{2}) = -\frac{1}{2}(\epsilon$

-2

(۶) $\lim_{x\to 2} \frac{\sqrt{2x+5}-3}{\sqrt{3x+1}-2} = \frac{0}{0}$

$\frac{d}{dx}(\sqrt{2x+5}) = \frac{2}{2\sqrt{2x+5}}$ $\Rightarrow x=2$ $\frac{2}{2\sqrt{14}} = \frac{2}{\frac{3}{2}\sqrt{14}}$ $= \frac{2}{\frac{3}{2}\cdot 4} = \frac{2}{6} = \frac{1}{3}$

$\frac{d}{dx}(\sqrt{3x+1}) = \frac{3}{2(3x+1)^{\frac{1}{2}}}$

(۷) $\lim_{x\to 1} \frac{\sqrt{2+\sqrt{3x}}-2}{\sqrt[3]{x}-1} = \frac{0}{0}$

$f(x) = \sqrt{2+\sqrt{3x}}$

$f'(x) = \frac{1}{2\sqrt{2+\sqrt{3x}}}\left(\frac{3}{2}\frac{1}{\sqrt{x}}\right)$ $\Rightarrow x=1$ $\Rightarrow \sqrt{2(1)+1} = 2$

$f'(1) = \frac{1}{4}\left(\frac{3}{2}\right) = \frac{1}{4} \times \frac{3}{2} = \frac{3}{8}$

$(\sqrt{x})' = \frac{1}{2x^{\frac{1}{2}}}$ $\Rightarrow x=1$ $\Rightarrow \frac{1}{2}$

$\frac{\frac{3}{8}}{\frac{1}{2}} = \frac{3}{8} \times 2 = \frac{3}{4}$

حل تمرین سری

(۸) $\lim_{x\to\pi} \frac{1+\cos^{۳} x}{\sin^{۲} x} = \frac{۰}{۰} = \frac{(1+\cos x)(1-\cos x+\cos^{۲} x)}{(1-\cos x)(1+\cos x)} = \frac{1-\cos x+\cos^{۲} x}{1-\cos x}$ $x=\pi$ ⇒

$\cos\pi = -1$
$1+(-1)^{۳} = ۰$
$\sin^{۲}\pi = ۰$
$\Big\} = \frac{۰}{۰}$

صورت $= 1-(-1)+(-1)^{۲} = 1+1+1 = ۳$ ($\cos\pi=-1$)
مخرج $= 1-(-1) = ۲$
$= \frac{۳}{۲}$ ✓

(۹) $\lim_{x\to\frac{\pi}{۴}} \frac{1-\tan x}{\sin x-\cos x} = \frac{۰}{۰}$ ⇒

$1-\frac{\sin}{\cos x} = \frac{\cos x-\sin x}{\cos x}$

$\frac{\cos x-\sin x}{\sin x-\cos x} = \sin x-\cos x = -(\cos x-\sin x)$

$\frac{\cos x-\sin x}{\cos x} \div -(\cos x-\sin x) = -\frac{1}{\cos x}$ ✓

$\tan\frac{\pi}{۴} = ۱ \quad 1-1 = ۰$

$\lim_{x\to\frac{\pi}{۴}} -\frac{1}{\cos x} = -\frac{1}{\frac{\sqrt{۲}}{۲}} = -\frac{۲}{\sqrt{۲}} = -\sqrt{۲}$

(۱۰) $\lim_{x\to\frac{۳\pi}{۴}} \frac{\tan^{۲} x-1}{\cos^{۲} x}$

$\tan^{۲} x-1 = \frac{1-\cos^{۲} x}{\cos^{۲} x}$

$1-\cos^{۲} x = \sin^{۲} x$

$\tan^{۲} x-1 = \frac{\sin^{۲} x}{\cos^{۲} x} = \tan^{۲} x$

$\frac{\tan^{۲} x-1}{\cos^{۲} x} = \tan^{۲} x = \frac{\sin^{۲} x}{\cos^{۴} x}$

$\sin^{۲}\frac{۳\pi}{۴} = \frac{\sqrt{۲}}{۲}$

$\cos^{۲}\frac{۳\pi}{۴} = -\frac{\sqrt{۲}}{۲}$

$\sin^{۲} x = \left(\frac{\sqrt{۲}}{۲}\right)^{۲} = \frac{1}{۲}$

$\cos^{۴} x = \left(-\frac{\sqrt{۲}}{۲}\right)^{۴} = \left(\frac{1}{۲}\right)^{۲} = \frac{1}{۴}$ ✓

$\frac{\frac{1}{۲}}{\frac{1}{۴}} = ۲$