① $\lim_{x\to 1} \frac{4x^2-7x+3}{5x^2-8x+3}$ تجزیه ⇒ شاخص صفر

$\frac{(4x-3)(x-1)}{(x-1)(5x-3)} = \frac{1}{2}$

② $\lim_{x\to 0} \frac{|4x-1|-|3x+1|}{x}$ ⇒ $\frac{-3x+1-3x-1}{x} = -6$

③ $\lim_{x\to 4} \frac{x-4}{\sqrt{x}-2}$ ⇒ $\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}-2)}$ ⇒ 4

④ $\lim_{x\to 2} \frac{x-\sqrt{3x}}{3x^2-x-5}$ ⇒ $\frac{x-\sqrt{3x}}{(x-2)(3x+5)} = \frac{\sqrt{x}(\sqrt{x}-\sqrt{3})}{(\sqrt{x}+\sqrt{3})(\sqrt{x}-\sqrt{3})(3x+5)}$

مزدوج ← تجزیه

$\frac{\sqrt{4}}{3\times 4+5} = \frac{1}{14}$

⑤ $\lim_{x\to 1} \frac{1-\sqrt{x}}{2-\sqrt{5-x}} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} \Rightarrow \times \frac{2+\sqrt{5-x}}{2+\sqrt{5-x}}$

$\Rightarrow \frac{(1-x)(4)}{2(-1+9)} = -2$ حالت $\frac{0}{0}$

$1-\sqrt{x}=0$ یا $1+\sqrt{x}\neq 0$ یا $x=1$ صفر می‌شود

$\sqrt{5-x}=2$ یا $x=1$

⑥ $\lim_{x\to 2} \frac{\sqrt{x^3+2}-2}{\sqrt[3]{5x+7}-3}$ × (ضرب في المرافق)

$\frac{\sqrt{x^3+2}+2}{\sqrt{x^3+2}+2}$ × $\frac{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}$

$= \frac{(x^3-8)}{(5x-10)} \times \frac{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}{\sqrt{x^3+2}+2}$

$= \frac{(x-2)(x^2+2x+4)}{5(x-2)} \times \frac{\sqrt[3]{(5x+7)^2}+3\sqrt[3]{5x+7}+9}{\sqrt{x^3+2}+2}$

$= \frac{12}{10} = \frac{6}{5}$

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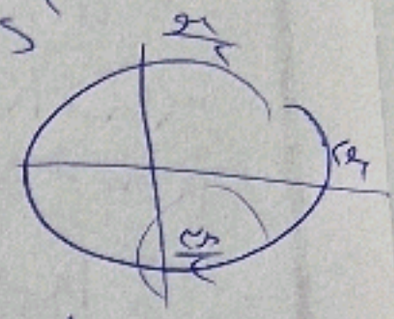
⑦ $\lim_{x\to 1} \frac{\sqrt[3]{x^3+\sqrt{x}-2}}{\sqrt[3]{x}-1}$ × (ضرب في المرافق)

$\frac{\sqrt[3]{x^3+\sqrt{x}+2}}{\sqrt[3]{x^3+\sqrt{x}+2}}$ × $\frac{\sqrt[3]{x^2}+\sqrt[3]{x}+1}{\sqrt[3]{x^2}+\sqrt[3]{x}+1}$

$= \frac{x^3+\sqrt{x}-2}{x-1} \times \frac{3}{4}$

$= \frac{(\sqrt{x}-1)(\sqrt{x}+2)}{(\sqrt{x}+1)(\sqrt{x}-1)} \times \frac{3}{4} = \frac{3}{8}$

(٨) $\lim_{x \to \frac{\pi}{2}} \frac{1+\cos^2 x}{\sin^2 x} \to \frac{1+\cos^2 x}{1-\cos^2}$



$\Rightarrow \frac{(1+\cos)(1+\cos^2 - \cos)}{(1-\cos)(1+\cos)} \Rightarrow \frac{1+1+1}{2} = \frac{3}{2}$

(٩) $\lim_{x \to \frac{\pi}{4}} \frac{1-\tan x}{\sin x - \cos x} \Rightarrow \frac{1-\frac{\sin}{\cos}}{\sin - \cos} \Rightarrow \frac{\frac{\cos - \sin}{\cos}}{\frac{(\cos - \sin)}{1}}$

$\frac{-1}{\cos x} = -\sqrt{2}$

(١٠) $\lim_{x \to \frac{\pi}{2}} \frac{\tan^2 x - 1}{\cos^2 x} \Rightarrow \frac{\sin^2 - \cos^2}{\cos^2}$

$\Rightarrow \frac{\cos^2 x - \sin^2 x}{1}$

$2 \to \frac{\pi}{2}$

$\frac{(-\sin - \cos)(\sin - \cos)}{\cos^2}$

$\Rightarrow \frac{-1}{\cos^2 x} = \frac{-1}{\frac{1}{2}} = -2$

(٢)