

$$\lim_{x \rightarrow 1} \frac{\sin x - \sqrt{x+3}}{x^2 - 4x + 3} = \frac{0}{0} \Rightarrow \frac{(x-1)(x-3)}{(x-1)(x+3)} = \frac{x-3}{x+3} \xrightarrow{x \rightarrow 1} \frac{1-3}{1+3} = \frac{-2}{4} = \boxed{-\frac{1}{2}}$$

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$$\lim_{x \rightarrow 0} \frac{\sqrt{x-1} - \sqrt{x+1}}{x} = \frac{0}{0} \xrightarrow{x \rightarrow 0} \frac{1 - \sqrt{x} - (\sqrt{x} + 1)}{x} = \frac{1 - \sqrt{x} - \sqrt{x} - 1}{x} = \frac{-2\sqrt{x}}{x} = \frac{-2}{\sqrt{x}} \rightarrow -\infty$$

$\boxed{-\infty}$

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$$\lim_{x \rightarrow 4} \frac{x-4}{\sqrt{x}-2} = \frac{0}{0} = \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(\sqrt{x}-2)} = \sqrt{x}+2 \xrightarrow{x \rightarrow 4} \sqrt{4}+2 = \boxed{4}$$

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$$\lim_{x \rightarrow 2} \frac{x - \sqrt{x}}{x^2 - x - 4} \times \frac{x + \sqrt{x}}{x + \sqrt{x}} = \frac{x^2 - x}{x^2 - x - 4 (x + \sqrt{x})} = \frac{x(x-1)}{(x-1)(x+4)(x+\sqrt{x})}$$

$$= \frac{x}{(x+4)(x+\sqrt{x})} = \frac{2}{2 \times 4} = \boxed{\frac{1}{4}}$$

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$$\lim_{n \rightarrow 1} \frac{1 - \sqrt{n}}{n - \sqrt{n}} = \frac{0}{0} \Rightarrow \frac{1 - \sqrt{n}}{n - \sqrt{n}} \times \frac{n + \sqrt{n}}{n + \sqrt{n}} = \frac{(1 - \sqrt{n})(n + \sqrt{n})}{n^2 - n}$$

$$= \frac{(1 - \sqrt{n})(n + \sqrt{n})}{n(n-1)} = \frac{(1 - \sqrt{n})(n + \sqrt{n})}{-n(1-n)} = \frac{(1 - \sqrt{n})(n + \sqrt{n})}{-n(1-n)(1+\sqrt{n})}$$

$$\xrightarrow{n \rightarrow 1} \frac{1 + \sqrt{1}}{-1(1+\sqrt{1})} = \frac{2}{-2} = \boxed{-1}$$

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$$\lim_{n \rightarrow \infty} \frac{\sqrt{\psi n + \xi} - \psi}{\sqrt{\omega n + \nu} - \psi} = \frac{0}{0} = \frac{\sqrt{\psi n + \xi} - \psi}{\sqrt{\omega n + \nu} - \psi} \times \frac{(\sqrt{\omega n + \nu} + \psi) + \sqrt{\omega n + \nu}}{(\sqrt{\omega n + \nu} + \psi) + \sqrt{\omega n + \nu}} \times \frac{\sqrt{\psi n + \xi} + \psi}{\sqrt{\psi n + \xi} + \psi}$$

$$= \frac{(\psi n + \xi - \psi^2)(\psi)}{(\omega n + \nu - \psi^2)(\psi)} = \frac{(\psi n - \psi^2)(\psi)}{(\omega n - \psi^2)(\psi)} = \frac{\psi(n - \psi) \times \psi}{\omega(n - \psi) \times \psi} = \boxed{\frac{\psi}{\omega}}$$

$$\lim_{n \rightarrow 1} \frac{\sqrt{\psi n + \sqrt{n}} - \psi}{\sqrt{n} - 1} = \frac{0}{0} = \frac{(\sqrt{\psi n + \sqrt{n}} - \psi)(\sqrt{n} + 1)}{(\sqrt{n} - 1)(\sqrt{n} + 1)} \times \frac{\sqrt{\psi n + \sqrt{n}} + \psi}{\sqrt{\psi n + \sqrt{n}} + \psi} = \frac{(\psi n + \sqrt{n} - \psi^2)(\psi)}{(n - 1)(\psi)}$$

$$= \frac{\psi((\psi n - \psi^2) + (\sqrt{n} - 1))}{\psi(n - 1)} = \frac{\psi(\psi(\sqrt{n} - 1)(\sqrt{n} + 1) + (\sqrt{n} - 1))}{\psi(\sqrt{n} - 1)(\sqrt{n} + 1)}$$

$$= \frac{\psi(\psi(\sqrt{n} + 1) + 1)}{\psi(\sqrt{n} + 1)} \xrightarrow{n \rightarrow 1} \frac{\psi(\psi + 1)}{\psi} = \boxed{\frac{\psi + 1}{1}}$$

$$\lim_{n \rightarrow \pi} \frac{1 + \cos^n n}{\sin^n n} = \frac{0}{0} = \frac{(1 + \cos n)(\cos^n n + 1 - \cos n)}{(1 - \cos^n n)} = \frac{(1 + \cos n)(\cos^n n + 1 - \cos n)}{(1 - \cos n)(1 + \cos n)}$$

$$= \frac{\cos^n n + 1 - \cos n}{1 - \cos n} \xrightarrow{n \rightarrow \pi} \frac{(-1)^n + 1 - (-1)}{1 - (-1)} = \frac{1 + 1 + 1}{1 + 1} = \boxed{\frac{3}{2}}$$

$$\lim_{n \rightarrow \frac{\pi}{2}} \frac{1 - \cos n}{\sin n - \cos n} = \frac{0}{0} = \frac{(1 - \frac{\sin n}{\cos n})}{\sin n - \cos n} = \frac{(\frac{\cos n - \sin n}{\cos n})}{\sin n - \cos n} = \frac{1}{\cos n} \cdot \frac{1}{-(\cos n - \sin n)} = \frac{1}{\cos n - \sin n}$$

$$\xrightarrow{n \rightarrow \frac{\pi}{2}} = \frac{1}{\cos \frac{\pi}{2} - \sin \frac{\pi}{2}} = \frac{1}{0 - 1} = -1 = \boxed{-\sqrt{1}}$$

$$\lim_{n \rightarrow \frac{\pi}{2}} \frac{\tan^n n - 1}{\cos^n n} = \frac{0}{0} = \frac{(\tan^n n - 1)(\tan^n n + 1)}{\cos^n n - \sin^n n} = \frac{(\tan^n n - 1)(\tan^n n + 1)}{(\cos^n n - \sin^n n)(\cos^n n + \sin^n n)}$$

$$= \frac{(\tan^n n - 1) \left(\frac{\sin^n n + \cos^n n}{\cos^n n} \right)}{(\cos^n n - \sin^n n)(\cos^n n + \sin^n n)} = \frac{(\tan^n n - 1) \left(\frac{1}{\cos^n n} \right)}{(\cos^n n - \sin^n n)} \xrightarrow{n \rightarrow \frac{\pi}{2}} \frac{(-1 - 1) \left(\frac{1}{-\sqrt{1}} \right)}{(-\sqrt{1} - \sqrt{1})}$$

$$= \frac{(-2) \cdot (-\sqrt{1})}{(-\sqrt{1})} = \boxed{-2}$$