[illegible]

یازدهم ریاضی B — تکلیف شماره ۴

سوال ۱

الف) $\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \to \begin{cases} x\neq 0 \\ x\neq 1 \end{cases} (I) \quad \frac{x-1}{x}-\frac{x}{x-1}\geq 0 \to \frac{(x-1)^2-x^2}{x(x-1)}\geq 0 \to \frac{1-2x}{x(x-1)}\geq 0$

x		0		$\frac{1}{2}$		1	
	$+$		$-$		$+$		$-$

$D=(-\infty,0)\cup[\frac{1}{2},1)\ (II)$

$$\boxed{I\cap II \Rightarrow D_f=(-\infty,0)\cup[\tfrac{1}{2},1)}$$

ب) $f(x)=\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \to \begin{cases} x\neq -1,0,1,-2 \quad (I) \\ \frac{3}{x-1}+\frac{1}{x+2}\neq 0 \to \frac{4x+5}{(x+2)(x-1)}\neq 0 \to x\neq -2,1,-\frac{5}{4} \quad (II) \end{cases}$

$$\boxed{I\cap II \Rightarrow D_f=\mathbb{R}-\{-2,-\tfrac{5}{4},-1,0,1\}}$$

سوال ۲

الف) $f(x)=\sqrt{((\frac{1}{3})^x-9)(32-4^x)} \to ((\tfrac{1}{3})^{\overset{-2}{x}}-9)(32-4^{\overset{\frac{5}{2}}{x}})\geq 0 \to$

x		-2		$\frac{5}{2}$	
	$+$		$-$		$+$

$$\boxed{D_f=(-\infty,-2]\cup[\tfrac{5}{2},+\infty)}$$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \quad \begin{cases} x-1\geq 0 \to x\geq 1 \quad (I) \\ \underbrace{\sqrt{y+1}}_{\text{نامنفی}}=3-\underbrace{\sqrt[4]{x-1}}_{\text{نامنفی}} \to \sqrt[4]{x-1}\leq 3 \to x-1\leq 81 \to x\leq 82 \quad (II) \end{cases}$

$$\boxed{I\cap II \Rightarrow D_f=[1,82]}$$

سوال ۳

$f(x)=\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1} \to \begin{cases} x^2-x-2>0 \to (x-2)(x+1)>0 \quad D=(-\infty,-1)\cup(2,+\infty)\ (I) \\ x^2-1\geq 0 \to D=(-\infty,-1]\cup[1,+\infty)\ (II) \\ \sqrt{x^2-1}\neq -1 \to \text{همیشه برقرار} \end{cases}$

x		-1		2	
	$+$		$-$		$+$

x		-1		1	
	$+$		$-$		$+$

$$\boxed{D_f=(-\infty,-1)\cup(2,+\infty)}$$

سوال ۴

$\sqrt{3+ax-x^2}\ ,\ [-2,b] \xrightarrow{x=-2} 3-2a-4=0 \to -2a-1=0 \to 2a=-1 \to \boxed{a=-\frac{1}{2}}$

$\to -x^2-\frac{1}{2}x+3 \to \Delta=\frac{1}{4}-4(-1)(3)=\frac{49}{4} \to \dfrac{\frac{1}{2}\pm\sqrt{\frac{49}{4}}}{-2} \Rightarrow \boxed{\begin{array}{l} x=-2 \\ x=\frac{3}{2}=b \end{array}}$

$$\boxed{a+b=-\tfrac{1}{2}+\tfrac{3}{2}=1}$$

سوال ۵

$f(x)=\begin{cases} 3x-2 & ; x\geq 1 \ (1) \\ 2x+3 & ; x<1 \ (2) \end{cases} \qquad g(x)=\sqrt{f(x)-x}$

$\begin{cases} (1): 3x-2-x=2x-2\geq 0 \\ \quad x\geq 1 \to D=[1,+\infty)\ (I) \\ (2): 2x+3-x=x+3\geq 0 \to x\geq -3 \xrightarrow[(2)]{\text{با توجه به شرط}} -3\leq x<1 \to D=[-3,1)\ (II) \end{cases}$

$$\boxed{I\cup II \Rightarrow D_f=[-3,+\infty)}$$

سوال ۶

$f(x)=\begin{cases} (a+1)(x+2) & ; x>1 \\ 3a+2x & ; x\leq 1 \end{cases} \qquad 2f(5)=f(-2)+a$

$2(a+1)(5+2)=3a+2(-2)+a$

$14a+14=4a-4$

$10a=-18$

$$\boxed{a=\frac{-18}{10}=\frac{-9}{5}}$$

$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$ $f(2-\sqrt{3}) + f(\underbrace{2+\sqrt{3}}_{t})$, $\frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$

$\sqrt{a} + \frac{1}{\sqrt{a}} + 2$

$\Rightarrow f(t) + f(\frac{1}{t}) = 2(\sqrt{t} + \frac{1}{\sqrt{t}}) + 4 \to 2(\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}}) + 4$

$2(\underbrace{\sqrt{2+\sqrt{3}} + \sqrt{2-\sqrt{3}}}_{a}) + 4$

$a^2 = (2-\sqrt{3}) + (2+\sqrt{3}) + 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} \to a^2 = 6 \quad a = \sqrt{6}$

$f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2a + 4 = \boxed{2\sqrt{6}+4}$

سوال ۸

$$\begin{cases} \times 2 & 2f(x) - 3f(-x) = 4x^2 - x \\ \times 3 & 2f(-x) - 3f(x) = \underbrace{4(-x)^2 - (-x)}_{4x^2+x} \end{cases} \Rightarrow \begin{cases} 4f(x) - 6f(-x) = 8x^2 - 2x \\ -9f(x) + 6f(-x) = 12x^2 + 3x \end{cases}$$

$-5f(x) = 20x^2 + x$

$f(x) = \boxed{\frac{-(20x^2+x)}{5} = -4x^2 - \frac{x}{5}}$

سوال ۹

$(x+2)f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$

$\underline{x=0}$ $2f(0) = 3m - 1 \quad \times -3$

$\underline{x=-2}$ $6f(0) = 16 + 2m + 3m - 1 \to 6f(0) = 5m + 15$

$\Big\} \to$ $-6f(0) = -9m + 3$, $6f(0) = 5m + 15$

$-4m + 18 = 0 \to 4m = 18 \to m = \frac{9}{2}$

$2f(0) = 3(\frac{9}{2}) - 1 = \frac{25}{2} \to \boxed{f(0) = \frac{25}{4}}$

سوال ۱۰

$f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x}$

$\underline{x=-1}$ $f(-1) + f(\frac{1}{-1}) = \frac{3(-1)^2 - 12(-1) + 3}{-1}$

$2f(-1) = -18 \to \boxed{f(-1) = -9}$