

الف) $\frac{x-1}{x} - \frac{x}{x-1} \geq 0 \quad x \neq 0 \quad x \neq 1$

$\frac{x-1}{x} \geq \frac{x}{x-1} \quad D = (-\infty, -1]$

$\frac{-x+1}{x(x-1)} \geq 0$
 $\frac{1}{x} - \frac{1}{x-1} \geq 0$
 $\frac{1}{x} \geq \frac{1}{x-1}$
 $\frac{1}{x} - \frac{1}{x-1} \geq 0$
 $\frac{1}{x(x-1)} \geq 0$

$\Rightarrow x+1 \neq 0 \quad x \neq -1 \quad \frac{x}{x-1} + \frac{1}{x+1} \neq 0 \quad \frac{x(x+1) + x-1}{(x-1)(x+1)} \neq 0$

$x \neq 0 \quad x-1 \neq 0 \quad x \neq 1 \quad \frac{x(x+1)}{(x-1)(x+1)} \neq 0 \quad x \neq -1 \quad x \neq \frac{-1}{x}$
 $x+1 \neq 0 \quad x \neq -1 \quad D = \mathbb{R} - \{-1, -\frac{1}{x}, 0\}$

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ب) $(x^2-1)(x-1) \geq 0 \quad D_f = (-\infty, -1] \cup [1, +\infty)$

$x-1 \geq 0 \rightarrow x \geq 1$
 $x^2-1 \geq 0 \rightarrow x \leq -1 \text{ or } x \geq 1$
 $D = \{x \mid x \leq -1 \text{ or } x \geq 1\}$

$\Rightarrow x-1 \geq 0 \quad x \geq 1$

$y+1 \geq 0 \quad y \geq -1$

$\sqrt{x-1} + \sqrt{y+1} = 2$

$D = \{(x, y) \mid x \geq 1, y \geq -1, \sqrt{x-1} + \sqrt{y+1} = 2\}$

$x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0 \rightarrow x > 2 \quad x < -1$

$\sqrt{x^2-1} + 1 > 0 \rightarrow \sqrt{x^2-1} > -1 \rightarrow x^2-1 > 0 \rightarrow x^2 > 1 \rightarrow |x| > 1 \rightarrow x \neq 0$

$D = (-\infty, -1) \cup (1, +\infty)$

$\sqrt{x^2 + ax + b} = -(x^2 - ax - b) \rightarrow x^2 - ax - b = 0$

$x = -2 \quad x = b$

$-2 \times b = -b$

$b = \frac{b}{-2}$

$-2 + b = a$

$a = -\frac{1}{2}$

$a + b = -\frac{1}{2} + \frac{b}{-2} = 1$

$f(x) - x \geq 0$

① $(3x-2) - x \geq 0 \quad 2x-2 \geq 0 \quad 2(x-1) \geq 0 \quad x \geq 1$

② $(2x+3) - x \geq 0 \quad x+3 \geq 0 \quad x \geq -3, \quad x < 1 \rightarrow [-3, 1)$

$D = [1, +\infty) \cup [-3, 1) \rightarrow [-3, +\infty)$

1

2

3

4

5

$$yf(w) = 15a + 15$$

$$f(-x) - a = 3a - 5 - a$$

$$15a + 15 = 3a - 5 - a$$

$$10a = -10$$

$$a = \frac{-9}{5}$$

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$$\sqrt{x+\sqrt{y}} + \frac{1}{\sqrt{x+\sqrt{y}}} + x + \sqrt{x-\sqrt{y}} + \frac{1}{\sqrt{x-\sqrt{y}}} + x \rightarrow$$

$$(\sqrt{x+\sqrt{y}} + \sqrt{x-\sqrt{y}}) + (\sqrt{x+\sqrt{y}} + \sqrt{x-\sqrt{y}}) + x \rightarrow x(\sqrt{x+\sqrt{y}} + \sqrt{x-\sqrt{y}}) + x$$

$$f(x-\sqrt{y}) + f(x+\sqrt{y}) = x\sqrt{y} + x$$

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$$yf(x) - xf(-x) = 5x^2 - x$$

$$xf(-x) - xf(x) = 5x^2 + x$$

$$5f(x) - 9f(-x) = 10x^2 - x \rightarrow -9f(x) = 10x^2 + x$$

$$9f(-x) - 9f(x) = 10x^2 + x$$

$$f(x) = -\frac{5x^2}{9} - \frac{x}{9}$$

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$$x=0 \rightarrow yf(0) = 5m-1 \rightarrow f(0) = \frac{5m-1}{y}$$

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$$x=-y \rightarrow f(0) = \frac{10+5m}{y}$$

$$\frac{10+5m}{y} = \frac{5m-1}{y} \rightarrow m = \frac{9}{5}$$

$$f(0) = \frac{18}{5}$$

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$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b \rightarrow a\left(x + \frac{1}{x}\right) + 2b \rightarrow \frac{a(x^2+1) + 2bx}{x}$$

$$\frac{ax^2 + 2bx + a}{x} = \frac{5x^2 - 14x + 5}{x} \quad a=5 \quad b=-9$$

$$f(-1) = 5(-1) - 9 \rightarrow f(-1) = -14$$

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