نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۴ ..... کلاس ..... دختر .....

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ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{4}{x-1}+\frac{1}{x+2}}$

$x\neq 1$ ← $x-1$ ، $x+2$ ← $x\neq -2$

$=\frac{\frac{x-2x-2}{x(x+1)}}{\frac{4x+4+x-1}{(x-1)(x+2)}}=\frac{(-x-2)(x-1)(x+2)}{x(x+1)(4x+5)}$

$-x-2\to -1$ ، $4x+5\to -\frac{5}{4}$

$Df=\mathbb{R}-\{0,-1,-\frac{5}{4},1,-2\}$

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ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

$\sqrt{y+1}=3-\sqrt[4]{x-1}$

$3\geq\sqrt[4]{x-1}\geq 0 \Rightarrow 0\leq x-1\leq 81$

$1\leq x\leq 82$

$Df=[1,82]$

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ب) $f(x)=\frac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1}$

$x^2-x-2>0 \to (x-2)(x+1)>0$

$\to x<-1$ یا $x>2$ (۱)

$\sqrt{x^2-1}+1\neq 0 \to \sqrt{x^2-1}\neq -1$ همیشه صدق می‌کند

$x^2-1\geq 0$

$x\leq -1$ یا $x\geq 1$ (۲)

(۱) ∩ (۲) →

$Df=(-\infty,-1)\cup(2,+\infty)$

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$\sqrt{3+ax-x^2}$ $Df=[-2,b]$ $a+b=?$

$3+ax-x^2\geq 0$

اگر $x=-2 \to 3-2a-4=0$

$-2a=1 \to a=-\frac{1}{2}$

$3+\frac{1}{2}x-x^2\geq 0$

$x=\frac{-\frac{1}{2}\pm\sqrt{\frac{1}{4}+24}}{-2}=\frac{-\frac{1}{2}\pm\frac{7}{2}}{-2}$

$\to -4$

$\to \frac{3}{2}=b$

$a=-\frac{1}{2}$

$b=\frac{3}{2}$

$a+b=1$

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$f(x)=\begin{cases}3x-2 & ;x\geq 1\\ 2x+3 & ;x<1\end{cases}$ $g(x)=\sqrt{f(x)-x}$ $Dg=?$

$\vdash\to f(x)\geq x$

اگر $x\geq 1$ $g(x)=\sqrt{3x-2-x}=\sqrt{2x-2} \Rightarrow 2x-2\geq 0$ $2x\geq 2$ $x\geq 1$ (۱)

اگر $x<1$ $g(x)=\sqrt{2x+3-x}=\sqrt{x+3} \Rightarrow x+3\geq 0$ $x\geq -3$ (۲)

(۱) ∪ (۲) $Dg=[-3,+\infty)$

$f(x) = \begin{cases} (a+1)(x+1) & ; x > 1 \\ 3a + 2x & ; x \leq 1 \end{cases}$

$f(3) \Rightarrow (a+1)(3) = 3a+3$

$f(-2) = 3a - 4$

$2f(3) = f(-2) + a$

$12a + 12 = 3a - 8 + a - 4$

$10a = -24$

$a = -12/10$

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$f(x) = \sqrt{x + \frac{1}{\sqrt{x}}} + 2$ $\quad f(2-\sqrt{3}) + f(2+\sqrt{3}) = ?$

$2-\sqrt{3}, \; 2+\sqrt{3} \longrightarrow$ معکوس

$(\sqrt{x} + \frac{1}{\sqrt{x}})^2 = x + \frac{1}{x} + 2$ $\quad f(x) = \sqrt{(\sqrt{x} + \frac{1}{\sqrt{x}})^2} + 2 \Rightarrow f(x) = \sqrt{x + \frac{1}{x} + 2} + 2$

$f(2-\sqrt{3}) = \sqrt{2 - \sqrt{3} + \frac{1}{2-\sqrt{3}} + 2} + 2 = \sqrt{2 - \sqrt{3} + 2 + \sqrt{3} + 2} + 2 = \sqrt{4} + 2$

$f(2+\sqrt{3}) = \sqrt{2 + \sqrt{3} + \frac{1}{2+\sqrt{3}} + 2} + 2 = \sqrt{2 + \sqrt{3} + 2 - \sqrt{3} + 2} + 2 = \sqrt{4} + 2$

$f(2-\sqrt{3}) + f(2+\sqrt{3}) = (\sqrt{4} + 2) = 4 + 2\sqrt{4}$

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$2f(x) - 3f(-x) = 4x^2 - x$ $\quad f(x) = ?$

if $x = -x \xrightarrow{x^3}$ $2f(-x) - 3f(x) = 4x^2 + x$

$\xrightarrow{\times 2}$ $2f(x) - 3f(-x) = 4x^2 - x$

$\oplus$ $\quad 4f(-x) - 9f(x) = 12x^2 + 3x$

$4f(x) - 9f(-x) = 8x^2 - 2x$

$-5f(x) = 20x^2 + x \Rightarrow f(x) = \frac{20x^2 + x}{-5}$

$f(x) = -4x^2 - \frac{x}{5}$

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$(x+2)\, f(x) - 3x f(x+2) = 4x^2 - mx + 3m - 1$ $\quad f(0) = ?$

if $x = 0$ $\quad 2f(0) = 3m - 1 \longrightarrow f(0) = \frac{3m-1}{2}$

if $x+2 = 0 \Rightarrow x = -2 \Rightarrow$ $4f(0) = 16 + 2m + 3m - 1 \Rightarrow 4f(0) = 5m + 15$

$f(0) = \frac{5m+15}{4}$

$\frac{3m-1}{2} = \frac{5m+15}{4} \Rightarrow 6m - 3 = 5m + 15 \Rightarrow m = 18$

if $f(0) = \frac{3m-1}{2}$ $\xrightarrow{m = \frac{9}{2}}$ $f(0) = \frac{(3 \times \frac{9}{2}) - 1}{2} = \frac{\frac{25}{2} - 2}{2} = \frac{\frac{21}{2}}{2} = \frac{21}{4}$

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$f(x) + f(\frac{1}{x}) = \frac{3x^2 - 12x + 3}{x}$ $\quad f(-1) = ?$ $\quad f(x) = ax + b$

$\frac{1}{2}$ نمره $\rightarrow$ if $x = -1 \Rightarrow f(-1) + f(-1) = \frac{3 + 12 + 3}{-1} \Rightarrow 2f(-1) = -18$

$f(-1) = -9$

$\frac{1}{2}$ نمره $\longrightarrow$ $f(x) = ax + b$, $f(\frac{1}{x}) = \frac{a}{x} + b$

$ax + b + \frac{a}{x} + b = \frac{3x^2 - 12x + 3}{x} \rightarrow 3\frac{(x^2 - 4x + 1)}{x} = \frac{ax^2 + 2bx + a}{x}$

$\frac{ax^2 + bx + a + b}{x}$

$a = 3 \Rightarrow f(x) = ax + b \rightarrow f(x) = 3x - 4$

$2b = -12 \Rightarrow b = -4$

$f(-1) = -3 - 4 = -9$

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