

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}}$

$$\frac{(x-1)^2 - x^2}{x(x-1)} \quad \text{و} \quad \text{Df} = (-\infty, 0) \cup \left[\frac{1}{2}, 1\right)$$

$$\frac{x^2+1-2x-x^2}{x^2-x} \quad \text{و} \quad \frac{1}{\frac{1}{2}} \quad \text{و} \quad \frac{1}{\frac{1}{2}}$$

$$\frac{1-2x}{x(x-1)} \quad \text{و} \quad \frac{1}{\frac{1}{2}}$$

ب) $f(x) = \frac{1}{x+1} - \frac{x}{x}$

$$x \neq -1 \quad \leftarrow \frac{x}{x-1} + \frac{1}{x+1}$$

$$\frac{x^2 - (x-1)(x+1)}{x(x+1)} = \frac{-x-1}{x(x+1)}$$

$$\text{Df} = \mathbb{R} - \left\{0, -1, -\frac{1}{2}, 1, 2\right\}$$

الف) $f(x) = \sqrt{\left(\frac{1}{x} - 4\right)(12 - x^2)}$

$$\left(\frac{1}{x} - 4\right)(12 - x^2) \geq 0$$

$$-x = 2 \quad x = 0 \quad \text{Df} = (-\infty, 2] \cup [12, +\infty)$$

$$x = -2 \quad x = 0 \quad x = 12$$

ب) $\sqrt{x-1} + \sqrt{y+1} = 2$

$$\sqrt{y+1} = 2 - \sqrt{x-1}$$

$$2 - \sqrt{x-1} \geq 0 \Rightarrow \sqrt{x-1} \leq 2$$

$$x-1 \leq 4 \quad x \leq 5$$

$$x-1 \geq 0 \Rightarrow x \geq 1$$

$$\text{Df} = [1, 5]$$

$f(x) = \log_f(x^2 - x - 2) \rightarrow x^2 - x - 2 = (x-2)(x+1)$

$$\frac{1}{\sqrt{x^2-1} + 1} = \frac{x^2 - x - 2}{\sqrt{x^2-1} + 1}$$

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$$\sqrt{x^2-1} \neq -1 \rightarrow \text{Df} = (-\infty, -1) \cup (2, +\infty)$$

① $x \geq 2$
② $x < -1$
① ∩ ②

$\sqrt{\mu + ax - x^2}$ $\text{Df} = [-a, b]$ $a+b = ?$

$$\mu + ax - x^2 \geq 0$$

if $x = -a \Rightarrow \mu - a^2 - \mu = 0$

$$a = -1 \quad \mu = -1 \quad \text{Df} = \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$a = \frac{1}{2} \quad \mu = -\frac{1}{4} \quad \text{Df} = \left[-\frac{1}{2}, \frac{1}{2}\right]$$

$$a+b = 1$$

$f(x) = \begin{cases} \mu x - 1 & ; x \geq 1 \\ x + \mu & ; x < 1 \end{cases}$ $g(x) = \sqrt{f(x) - x}$ $\text{Dg} = ?$

if $x \geq 1$ $g(x) = \sqrt{\mu x - 1 - x} = \sqrt{(\mu-1)x - 1}$ $(\mu-1)x - 1 \geq 0$ ①

if $x < 1$ $g(x) = \sqrt{x + \mu - x} = \sqrt{\mu}$ $\mu \geq 0$ ②

$$\text{Dg} = [-\mu, +\infty)$$

$$f(x) = \sqrt{(a+1)(x+r)} \quad ; x > 1$$

$$\sqrt{a+1} \sqrt{x} \quad ; x > 1$$

$$f(x) \Rightarrow (a+1)(v) = va+v$$

$$f(-r) = \sqrt{a} - r$$

$$rf(x) = f(-r) + a$$

$$1(a+1) = \sqrt{a} - r + a \quad \Leftrightarrow -r$$

$$\frac{1 \cdot a = -11}{a = -1,1}$$

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$$f(x) = \sqrt{x + \frac{1}{x}} + r \quad f(r-\sqrt{r}) + f(r+\sqrt{r}) = ?$$

$$\left(\sqrt{x + \frac{1}{x}}\right)^2 = x + \frac{1}{x} + r \quad f(x) = \sqrt{\left(\sqrt{x + \frac{1}{x}}\right)^2 + r} \Rightarrow f(x) = \sqrt{x + \frac{1}{x} + r} + r$$

$$f(r-\sqrt{r}) = \sqrt{r-\sqrt{r} + \frac{1}{r-\sqrt{r}} + r} + r = \sqrt{r - \sqrt{r} + r + \sqrt{r} + r} + r = \sqrt{4+r}$$

$$f(r+\sqrt{r}) = \sqrt{r+\sqrt{r} + \frac{1}{r+\sqrt{r}} + r} + r = \sqrt{r + \sqrt{r} + r - \sqrt{r} + r} + r = \sqrt{4+r}$$

$$f(r-\sqrt{r}) + f(r+\sqrt{r}) = \sqrt{4+r} = \boxed{r + \sqrt{4+r}}$$

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$$rf(x) - rf(-x) = fax - a$$

$$\text{if } x = -x \rightarrow \begin{cases} rf(-x) - rf(x) = fax + a \\ rf(x) - rf(-x) = fax - a \end{cases}$$

$$\begin{cases} 4f(-x) - 4f(x) = 4fax + 4a \\ 4f(x) - 4f(-x) = 4fax - 4a \end{cases}$$

$$-2f(x) = r \cdot fax + a \Rightarrow f(x) = \frac{r \cdot fax + a}{-2}$$

$$f(x) = ?$$

$$\boxed{f(x) = -fax - \frac{a}{2}}$$

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$$(x+r) f(x) - rx f(x+r) = fax - mx + r^{m-1}$$

$$f(x) = ?$$

$$\text{if } x = 0, \quad rf(x) = r^{m-1} \rightarrow f(x) = \frac{r^{m-1}}{r}$$

$$\text{if } x+r = 0 \Rightarrow x = -r \Rightarrow 4f(x) = r(4+r) + r^{m-1} \Rightarrow 4f(x) = \delta m + 18$$

$$f(x) = \frac{\delta m + 18}{4}$$

$$\frac{r^{m-1}}{r} = \frac{\delta m + 18}{4} \Rightarrow 4r^m - r = \delta m + 18 \Rightarrow \begin{cases} m=11 \\ m=9 \end{cases}$$

$$\text{if } f(x) = \frac{r^{m-1}}{r} \xrightarrow{m=9} f(x) = \frac{(r^9)^{m-1}}{r} = \frac{r^{9m-9}}{r} = \frac{r^{9m-10}}{r} = \frac{r^{9m-11}}{r} = \frac{r^{9m-12}}{r} = \frac{r^{9m-13}}{r} = \frac{r^{9m-14}}{r} = \frac{r^{9m-15}}{r} = \frac{r^{9m-16}}{r} = \frac{r^{9m-17}}{r} = \frac{r^{9m-18}}{r} = \frac{r^{9m-19}}{r} = \frac{r^{9m-20}}{r} = \frac{r^{9m-21}}{r} = \frac{r^{9m-22}}{r} = \frac{r^{9m-23}}{r} = \frac{r^{9m-24}}{r} = \frac{r^{9m-25}}{r} = \frac{r^{9m-26}}{r} = \frac{r^{9m-27}}{r} = \frac{r^{9m-28}}{r} = \frac{r^{9m-29}}{r} = \frac{r^{9m-30}}{r} = \frac{r^{9m-31}}{r} = \frac{r^{9m-32}}{r} = \frac{r^{9m-33}}{r} = \frac{r^{9m-34}}{r} = \frac{r^{9m-35}}{r} = \frac{r^{9m-36}}{r} = \frac{r^{9m-37}}{r} = \frac{r^{9m-38}}{r} = \frac{r^{9m-39}}{r} = \frac{r^{9m-40}}{r} = \frac{r^{9m-41}}{r} = \frac{r^{9m-42}}{r} = \frac{r^{9m-43}}{r} = \frac{r^{9m-44}}{r} = \frac{r^{9m-45}}{r} = \frac{r^{9m-46}}{r} = \frac{r^{9m-47}}{r} = \frac{r^{9m-48}}{r} = \frac{r^{9m-49}}{r} = \frac{r^{9m-50}}{r} = \frac{r^{9m-51}}{r} = \frac{r^{9m-52}}{r} = \frac{r^{9m-53}}{r} = \frac{r^{9m-54}}{r} = \frac{r^{9m-55}}{r} = \frac{r^{9m-56}}{r} = \frac{r^{9m-57}}{r} = \frac{r^{9m-58}}{r} = \frac{r^{9m-59}}{r} = \frac{r^{9m-60}}{r} = \frac{r^{9m-61}}{r} = \frac{r^{9m-62}}{r} = \frac{r^{9m-63}}{r} = \frac{r^{9m-64}}{r} = \frac{r^{9m-65}}{r} = \frac{r^{9m-66}}{r} = \frac{r^{9m-67}}{r} = \frac{r^{9m-68}}{r} = \frac{r^{9m-69}}{r} = \frac{r^{9m-70}}{r} = \frac{r^{9m-71}}{r} = \frac{r^{9m-72}}{r} = \frac{r^{9m-73}}{r} = \frac{r^{9m-74}}{r} = \frac{r^{9m-75}}{r} = \frac{r^{9m-76}}{r} = \frac{r^{9m-77}}{r} = \frac{r^{9m-78}}{r} = \frac{r^{9m-79}}{r} = \frac{r^{9m-80}}{r} = \frac{r^{9m-81}}{r} = \frac{r^{9m-82}}{r} = \frac{r^{9m-83}}{r} = \frac{r^{9m-84}}{r} = \frac{r^{9m-85}}{r} = \frac{r^{9m-86}}{r} = \frac{r^{9m-87}}{r} = \frac{r^{9m-88}}{r} = \frac{r^{9m-89}}{r} = \frac{r^{9m-90}}{r} = \frac{r^{9m-91}}{r} = \frac{r^{9m-92}}{r} = \frac{r^{9m-93}}{r} = \frac{r^{9m-94}}{r} = \frac{r^{9m-95}}{r} = \frac{r^{9m-96}}{r} = \frac{r^{9m-97}}{r} = \frac{r^{9m-98}}{r} = \frac{r^{9m-99}}{r} = \frac{r^{9m-100}}{r}$$

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$$f(x) + f\left(\frac{1}{x}\right) = \frac{rx^r - 1}{x+r}$$

$$f(-1) = ?$$

$$f(x) = ax + b$$

$$\text{if } x = -1 \Rightarrow f(-1) + f(-1) = \frac{r+1+r}{-1} \Rightarrow rf(-1) = -11$$

$$f(x) = ax + b, \quad f\left(\frac{1}{x}\right) = \frac{a}{x} + b \quad \boxed{f(-1) = -9}$$

$$\frac{ax+b + \frac{a}{x} + b}{x} = \frac{rx^r - 1}{x+r} \Rightarrow \frac{ax^2 + bx + a + bx^2 + b^2x}{x^2} = \frac{ax^2 + bx + a}{x}$$

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$$a = r$$

$$rb = -1r \Rightarrow b = -1$$

$$\Rightarrow f(x) = ax + b \rightarrow f(x) = rx - 1$$

$$f(-1) = -r - 1 = -9$$