

الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0 \rightarrow \frac{x-1}{x} - \frac{x}{x-1} \geq 0 \rightarrow \frac{1-x^2}{x(x-1)} \geq 0 \rightarrow \frac{1}{x} - \frac{1}{x-1} \geq 0 \rightarrow D_f = (-\infty, 0) \cup [\frac{1}{2}, +\infty)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{3}{x-1} + \frac{1}{x+2}} = \frac{\frac{-2-x}{x(x+1)}}{\frac{x^2+x-2}{x^2+x-2}} \rightarrow x = -1, 0$
 $\rightarrow x = -\frac{5}{2} \rightarrow D_f = \mathbb{R} - \{0, -1, -\frac{5}{2}, 1, -\frac{5}{2}\}$

الف) $f(x) = \sqrt{(\frac{1}{3}x - 9)(32 - x^2)} \geq 0 \rightarrow (\frac{1}{3}x - 9)(32 - x^2) \geq 0 \rightarrow \frac{-2}{3} \leq x \leq 4 \rightarrow D_f = (-\infty, -\frac{2}{3}] \cup [4, +\infty)$

ب) $f(x) = \sqrt{x-1} + \sqrt{y+1} = 3$

$\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow 3 - \sqrt{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow 0 \leq x-1 \leq 9 \rightarrow 1 \leq x \leq 10$
 $\textcircled{1} \sqrt{x-1} \geq 0 \rightarrow x \geq 1$
 $\textcircled{2} x-1 \leq 9 \rightarrow x \leq 10$
 $D_f = \textcircled{1} \cap \textcircled{2} = [1, 10]$

$f(x) = \frac{\log_{\frac{1}{2}}(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \geq 0 \rightarrow \frac{-1}{2} \leq x \leq 2$

$D_f = \textcircled{1} \cap \textcircled{2} = (-\infty, -1) \cup (2, +\infty)$

$\sqrt{x^2 - 1} \geq 0 \rightarrow x^2 \geq 1 \rightarrow x \geq 1, x \leq -1$

$3 + ax - x^2 \geq 0$

$x = -2 \rightarrow 3 - 2a - 4 = 0$
 $a = -\frac{1}{2}$

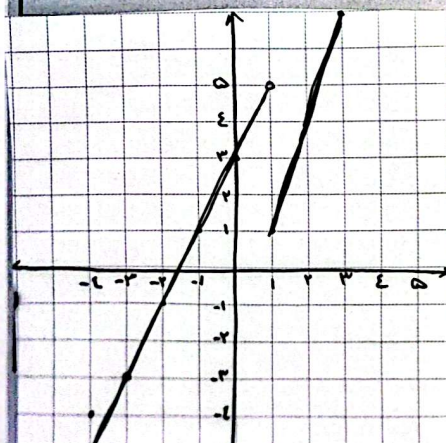
$3 - \frac{1}{2}x - x^2 = 0 \rightarrow a + b = \frac{3}{2} - \frac{1}{2} = 1$

$x = -2, \frac{3}{2} \rightarrow b = \frac{3}{2}$

$f(x) = \begin{cases} x^2 - 2 & ; x \geq 1 \\ x^2 + 3 & ; x < 1 \end{cases}$

$g(x) = \sqrt{f(x) - x} \geq 0 \rightarrow f(x) - x \geq 0 \rightarrow f(x) \geq x$

$D_f = [-3, +\infty)$



$$rf(a) = f(-r) + a$$

$$r(a+1)(a+r) = ra + r(-r) + a$$

$$18a + 18 = ra - r$$

$$10a = -18$$

$$a = -1.8 = -\frac{9}{5}$$

$$\begin{cases} (a+1)(n+r); x > 1 \\ ra + rn; x \leq 1 \end{cases}$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + r \rightarrow f(r - \sqrt{r}) + f(r + \sqrt{r}) = ?$$

$$\sqrt{r - \sqrt{r}} + \frac{1}{\sqrt{r - \sqrt{r}}} + r + \sqrt{r + \sqrt{r}} + \frac{1}{\sqrt{r + \sqrt{r}}} + r \xrightarrow{\sqrt{r - \sqrt{r}}} r + r\sqrt{r + \sqrt{r}} + r\sqrt{r - \sqrt{r}}$$

$$\frac{1}{\sqrt{r - \sqrt{r}}} \times \frac{\sqrt{r + \sqrt{r}}}{\sqrt{r + \sqrt{r}}} = \frac{\sqrt{r + \sqrt{r}}}{1} = \sqrt{r + \sqrt{r}}$$

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$$\begin{cases} rf(r) - rf(-r) = rn^2 - n \\ rf(-r) - rf(r) = rn^2 + n \end{cases} \rightarrow \begin{cases} rf(r) - rf(-r) = rn^2 - n \\ rf(-r) - rf(r) = rn^2 + n \end{cases} \xrightarrow{+} -2f(n) = rn^2 + n \rightarrow$$

$$f(n) = \frac{rn^2 + n}{-2}$$

$$f(n) = -\frac{rn^2}{2} - \frac{n}{2}$$

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$$-rx^2 f(x+r) + (x+r)f(x) = rx^2 - mx + r^2m - 1$$

$$\begin{cases} x = -r \rightarrow rf(0) = 18 + 18m \\ x = 0 \rightarrow rf(0) = r^2m - 1 \end{cases} \xrightarrow{rf(0)} \begin{cases} 18 + 18m = 9m - 1 \rightarrow 9m = -19 \rightarrow m = -\frac{19}{9} \end{cases}$$

$$x = 0 \rightarrow rf(0) = r^2m - 1 \xrightarrow{rf(0)} -19 + 9m$$

$$rf(0) = r\left(\frac{9}{r}\right) - \frac{r}{r} = \frac{9}{r} \rightarrow f(0) = \frac{\frac{9}{r}}{\frac{r}{1}} = \frac{9}{r^2}$$

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$$f(x) + f\left(\frac{1}{x}\right) = \frac{rx^2 - 18x + r}{x} \xrightarrow{f(-1)} x = -1 : f(-1) + f(-1) = \frac{r + 18 + r}{-1} = -18 \rightarrow rf(-1) = -18$$

$$f(-1) = -\frac{18}{r} = -9$$

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