تکلیف شماره ۴ ایدا محمدزاد یازدهم ریاضی B

$x \neq 0 , x \neq 1$

۱)

الف) $f(x)=\sqrt{\dfrac{x-1}{x}-\dfrac{x}{x-1}} \geq 0$

II $x^2-x \neq 0 \Rightarrow x(x-1)\neq 0 \to x\neq 0 ,\ x \neq 1$

$\dfrac{(x-1)\,x-1}{(x-1)\,x}-\dfrac{x(x)}{x-1(x)} \geq 0 \to \dfrac{\cancel{x^2}-2x+1-\cancel{x^2}}{x^2-x} \geq 0 \Rightarrow \dfrac{-2x+1}{x^2-x}$

$-2x+1=0 \to -2x=-1 \to x=\frac{1}{2} \leftarrow$ صورت

$$\begin{array}{ccccccc} & 0 & & \frac{1}{2} & & 1 & \\ + & \circ & - & \bullet & + & \circ & - \end{array}$$

$\boxed{D_y=(-\infty,0)\cup[\frac{1}{2},1)}$

ب) $f(x)=\dfrac{\dfrac{1}{x+1}-\dfrac{2}{x}}{\dfrac{3}{x-1}+\dfrac{1}{x+2}}$

مخرج $\to \dfrac{(x+2)\,3}{(x+2)\,x-1}+\dfrac{1(x-1)}{x+2(x-1)} \to \dfrac{3x+6+x-1}{x^2-x+2x-2}=\dfrac{4x+5}{x^2+x-2}\neq 0$

$4x+5=0 \to 4x=-5 \to \boxed{x=\frac{-5}{4}}$

$\boxed{x=1}$ و $\boxed{x=-2}$

صورت $\to \dfrac{(x)\,1}{(x)\,x+1}-\dfrac{2(x+1)}{x(x+1)} \to \dfrac{x-2x-1}{x^2-x}$

$x^2-x\neq 0 \to x(x-1)\neq 0 \to x\neq 0 ,\ x\neq 1$

$\boxed{D_y=R-\{-2,-\frac{5}{4},0,1\}}$

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۲)

الف) $f(x)=\sqrt{\left[(\frac{1}{3})^x-9\right](32-4^x)} \geq 0$

$4^x=32 \Rightarrow 2^{2x}=2^5 \Rightarrow 2x=5 \Rightarrow x=\frac{5}{2}$

$(\frac{1}{3})^x=9 \Rightarrow 3^{-x}=3^2 \Rightarrow -x=2 \to x=-2$

$$\begin{array}{ccccc} & -2 & & \frac{5}{2} & \\ + & \bullet & - & \bullet & + \end{array}$$

$\boxed{D_y=(-\infty,-2]\cup[\frac{5}{2},+\infty)}$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3$

I $\sqrt[4]{x-1} \to x-1 \geq 0 \to x \geq 1$

II $\sqrt{y+1} \to y+1 \geq 0 \Rightarrow y \geq -1$

$\boxed{D_y=\{(x,y)\in R \mid x\geq 1 ,\ y \geq -1\}}$

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۳)

$f(x)=\dfrac{\log_4^{(x^2-x-2)}}{\sqrt{x^2-1}+1}$

$$\begin{array}{ccccc} & -1 & & 2 & \\ + & \circ & - & \circ & + \end{array}$$

I $x^2-x-2>0 \Rightarrow \underset{2}{(x-2)}\ \underset{-1}{(x+1)}>0$

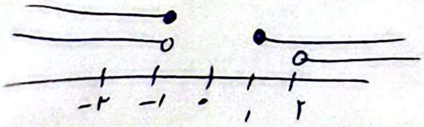
$2<x$ یا $x<-1$

II $x^2-1 \geq 0 \to x^2 \geq 1 \Rightarrow x \geq 1$ یا $x \leq -1$

III $\sqrt{x^2-1}+1 \neq 0 \Rightarrow \sqrt{x^2-1}\neq -1$

هیچ وقت صفر نمی‌شود.

$I \cap II$



$\boxed{D_y=(-\infty,-1)\cup(2,+\infty)}$

(۴

$\sqrt{۳+ax-x^۲}$ $\quad ۳+ax-x^۲\geqslant ۰$ $\quad$ دامنه $[-۲,b]$ $\quad x=\frac{-a\pm\sqrt{a^۲+۱۲}}{۲}$

$a+b=?$ $\quad$ ریشه‌های معادله $\quad ۳+ax-x^۲=۰$ $\quad x=\frac{-b\pm\sqrt{\Delta}}{۲a}$

$x=-۲ \to ۳+a(-۲)-(-۲)^۲=۰ \Rightarrow a(-۲)-۱=۰ \to a=-\frac{۱}{۲}$

گزینه

$a+b=\frac{۳}{۲}-\frac{۱}{۲}=۱$

$b=\frac{a+\sqrt{۱۲+a^۲}}{۲}=\frac{\sqrt{۱۲+\frac{۱}{۴}}-\frac{۱}{۲}}{۲}=\frac{\sqrt{\frac{۴۹}{۴}}-\frac{۱}{۲}}{۲}=\frac{۳}{۲}=۱/۵$

(۵

$f(x)=\begin{cases} ۳x-۲ \;; & x\geqslant ۱ \\ ۲x+۳ \;; & x<۱ \end{cases}$ $\quad g(x)=\sqrt{f(x)-x}\geqslant ۰ \Rightarrow f(x)\geqslant x$

شرط

$f(x)-x=(۲x+۳)-x=x+۳\geqslant ۰ \Rightarrow x\geqslant -۳ \Rightarrow -۳\leqslant x<۱$

$f(x)=۳x-۲ \to f(x)-x=(۳x-۲)-x=۲x-۲\geqslant ۰ \Rightarrow x\geqslant ۱$

$D_g=[-۳,۱)\cup[۱,+\infty)$

$=[-۳,+\infty)$

(۶

$f(x)=\begin{cases} (a+۱)(x+۲) \;; & x>۱ \\ ۳a+۲x \;; & x\leqslant ۱ \end{cases}$ $\quad ۲f(۵)=f(-۲)+a \quad a=?$

I $\quad f(۵)=(a+۱)(۵+۲)=(a+۱)۷=۷a+۷$

$۲f(۵)=۲(۷a+۷)=۱۴a+۱۴$

II $\quad f(-۲)=۳a+۲(-۲)=۳a-۴$

$f(-۲)$

$۲f(۵)=f(-۲)+a \Rightarrow$

$۱۴a+۱۴=\underbrace{۳a-۴+a}_{۴a} \Rightarrow$

$\underbrace{۱۴a-۴a}_{۱۰a}=\underbrace{-۴-۱۴}_{-۱۸} \to a=\frac{-۱۸}{۱۰}=-\frac{۹}{۵}$

$-۱/۸$

(۷

$f(x)=\sqrt{x}+\frac{۱}{\sqrt{x}}+۲$ $\quad f(۲-\sqrt{۳})+f(۲+\sqrt{۳})=?$

$\sqrt{۲-\sqrt{۳}}+\frac{۱}{\sqrt{۲-\sqrt{۳}}}\times\frac{\sqrt{۲+\sqrt{۳}}}{\sqrt{۲+\sqrt{۳}}}+۲ \to \sqrt{۲-\sqrt{۳}}+\sqrt{۲+\sqrt{۳}}+۲$

$\sqrt{۲+\sqrt{۳}}+\frac{۱}{\sqrt{۲+\sqrt{۳}}}\times\frac{\sqrt{۲-\sqrt{۳}}}{\sqrt{۲-\sqrt{۳}}}+۲ \to \sqrt{۲+\sqrt{۳}}+\sqrt{۲-\sqrt{۳}}+۲$

$\sqrt{۲-\sqrt{۳}}+\sqrt{۲+\sqrt{۳}}+۲+\sqrt{۲+\sqrt{۳}}+\sqrt{۲-\sqrt{۳}}+۲=۴+۲\sqrt{۲-\sqrt{۳}}+۲\sqrt{۲+\sqrt{۳}}$

$=۴+۲\left(\sqrt{۲-\sqrt{۳}}+\sqrt{۲+\sqrt{۳}}\right)$

$t^۲=۲+\sqrt{۳}+۲-\sqrt{۳}+۲ \to t=\sqrt{۶}$

گزینه ۱ $\to ۴+۲\sqrt{۶}$

$$2f(x) - 3f(-x) = 4x^2 - x \qquad f(x) = ? \qquad (8$$

$$\begin{aligned} 2\times 2f(x) - 3f(-x) &= 4x^2 - x \\ 3\times -3f(x) + 2f(-x) &= -4x^2 + x \end{aligned} \Rightarrow \begin{cases} 4f(x) - \cancel{6f(-x)} = 8x^2 - 2x \\ -9f(x) + \cancel{6f(-x)} = -12x^2 + 3x \end{cases}$$

$$-5f(x) = -4x^2 + x \Rightarrow f(x) = \frac{-4x^2 + x}{-5} =$$

$$\frac{x(-4x+1)}{-5} = \boxed{\frac{4}{5}x^2 - \frac{1}{5}x}$$

ج

$$(x+2)f(x) - 3xf(x+2) = 4x^2 - mx + 3m - 1 \qquad f(0) = ? \qquad (9$$

$$x = 0 \longrightarrow 2f(0) = 3m - 1$$

$$x = -2 \longrightarrow 6f(0) = 16 + 2m + 3m - 1$$

$$\Rightarrow \begin{cases} 2f(0) = 3m - 1 \longrightarrow f(0) = \dfrac{3m-1}{2} \\ 6f(0) = 5m + 15 \longrightarrow f(0) = \dfrac{5m+15}{6} \end{cases}$$

$$\frac{3m-1}{2} = \frac{5m+15}{6} \Rightarrow 9m - 3 = 5m + 15 \longrightarrow 4m = 18 \Rightarrow m = \frac{18}{4} = \frac{9}{2}$$

$$\frac{3m-1}{2} \longrightarrow \frac{3 \times \frac{9}{2} - 1}{2} = \frac{25}{4} \longrightarrow$$ ج

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \qquad (10$$

$$f(-1) = ?$$

I جاگذاری

$$f(x) + f\left(\frac{1}{x}\right) = ax + b + \frac{a}{x} + b = \left(\frac{1}{x} + x\right)a + 2b$$

$$\frac{3x^2 - 12x + 3}{x} = 3x - 12 + \frac{3}{x} \qquad f(x) = 3x - 6 \longrightarrow f(-1) = \underbrace{3(-1)}_{-3} - 6 = \boxed{-9}$$

$$\left(\frac{1}{x} + x\right)a + 2b = 3x - 12 + \frac{3}{x}$$

$$2b = -12 \Rightarrow \boxed{b = -6}$$

پاسخ