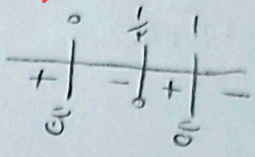
(یازدهم دفتر B) (فاطمه شکرانی) (تکلیف‌ی)

۲۰

۱) الف) $f(x)=\sqrt{\frac{x-1}{x}-\frac{x}{x-1}}$

$\frac{x-1}{x}-\frac{x}{x-1}\geq 0$

$\frac{x^2+1-2x-x^2}{x^2-x}=\frac{1-2x}{x(x-1)}\geq 0$



$D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $f(x)=\frac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}}$

$x\neq 0,-1,1,-2$

$\frac{3}{x-1}+\frac{1}{x+2}\neq 0 \quad \frac{3x+6+x-1}{(x-1)(x+2)}=\frac{4x+5}{(x-1)(x+2)} \rightarrow x\neq -\frac{5}{4}$

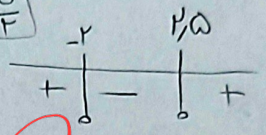
$D_f=R-\{-2,-\frac{5}{4},-1,0,1\}$

۲) الف) $f(x)=\sqrt{\left[\left(\frac{1}{2}\right)^x-9\right](3^{2x}-4^x)}$

$\left(\left(\frac{1}{2}\right)^x-9\right)(3^{2x}-4^x)\geq 0$

$3^{-2x}-9=0 \quad 3^{-2x}=3^2 \quad x=-2$

$2^{5x}=3^{2x}=2^{4x} \quad x=0$



$D_f=R-(-2,2,0)$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \quad x-1\geq 0 \quad x\geq 1$

$\sqrt{y+1}=3-\sqrt[4]{x-1}\geq 0$

$3\geq\sqrt[4]{x-1}$

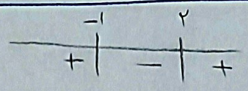
$81\geq x-1 \rightarrow 82\geq x$

$D_f=[1,82]$

۳) $f(x)=\frac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1}$

$\sqrt{x^2-1}+1$ همیشه >0

$(x-2)(x+1)>0$



$\rightarrow(-\infty,-1)\cup(2,+\infty)$ ①

$x^2-1\geq 0 \quad x^2\geq 1$

$x\geq 1$ $x\leq -1$ ②

$①\cap②=(-\infty,-1]\cup(2,+\infty)$

۴) $f(x)=\sqrt{3+ax-x^2}$

$D_f=[-2,b]$

$a+b=?$

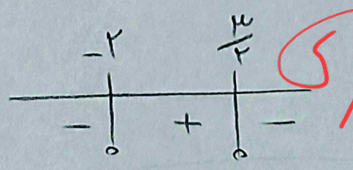
$-x^2+ax+3\geq 0$

$-(-2)^2+a(-2)+3=0$

$-4-2a+3=0 \rightarrow -1=2a \rightarrow a=-\frac{1}{2}$

$-x^2-\frac{1}{2}x+3\geq 0$

$\Delta=\frac{1}{4}-4(-1)(3)=\frac{49}{4}$



$\frac{\frac{1}{2}\pm\frac{7}{2}}{-2}$ $\begin{matrix}-2\\ \frac{3}{2}\end{matrix}$

$\left(-\frac{1}{2}\right)+\left(\frac{3}{2}\right)=\frac{2}{2}=1$

۵) $f(x)=\begin{cases}2x-2 & ; x\geq 1\\ 2x+3 & ; x<1\end{cases}$

$g(x)=\sqrt{f(x)-x}$ $f(x)-x\geq 0$

برای اعداد بزرگتر مساوی ۱ همواره برقرار است $f(x)\geq x$

و برای اعداد کوچکتر از ۱ از ضابطه ۲ برقرار است

$\rightarrow[-3,+\infty)$

$x<1 \rightarrow \sqrt{2x+3-x}=\sqrt{x+3}$ $x+3\geq 0$

$\boxed{-3\leq x}$

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$$f(x)=\begin{cases}(a+1)(x+2) & ; x>1\\ x^2a+2x & ; x\le 1\end{cases}$$

$2f(a)=f(-2)+a$

$a=?$

$2(a+1)(7)=2a+2(-2)+a$

$14a+14=4a-4$

$10a=-18$

$a=\frac{-18}{10}$

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$f(x)=\sqrt{x}+\frac{1}{\sqrt{x}}+2$ =

$f(2-\sqrt{3})+f(2+\sqrt{3})=?$

$\sqrt{2-\sqrt{3}}+\sqrt{\frac{1}{2-\sqrt{3}}}+2+\sqrt{2+\sqrt{3}}+\sqrt{\frac{1}{2+\sqrt{3}}}+2=4+2\sqrt{2-\sqrt{3}}+2\sqrt{2+\sqrt{3}}$

$\sqrt{2+\sqrt{3}}$, $\sqrt{2-\sqrt{3}}$

$=4+\sqrt{2}\left(\sqrt{4-2\sqrt{3}}+\sqrt{4+2\sqrt{3}}\right)=$

$4+\sqrt{2}\left(\sqrt{3}-1+\sqrt{3}+1\right)=2\sqrt{6}+4$

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$2f(x)-3f(-x)=4x^2-x$

$f(x)=?$

$2f(x)-3f(-x)=4x^2-x \quad \leftarrow (x)$

$3\times\ 2f(-x)-3f(x)=4x^2+x \quad \leftarrow (-x)$

$4f(x)=9f(-x)=8x^2-2x$

$-9f(-x)+9f(x)=12x^2+3x$

$-5f(x)=20x^2+x \rightarrow f(x)=-4x^2-\frac{1}{5}x$

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$(m+2)f(x)-2xf(x+2)=4x^2-mx+3m-1$

$f(0)=?$

$x=0 \rightarrow (2)f(0)=3m-1 \rightarrow f(0)=\frac{3m-1}{2}$

$x=-2 \rightarrow (6)f(0)=19+5m-1 \rightarrow f(0)=\frac{10+5m}{6}$

$\frac{3m-1}{2}=\frac{10+5m}{6}$

$9m-3=10+5m$

$m=\frac{13}{4}=\frac{9}{4}$ $\quad f(0)=\frac{3\frac{13}{4}-1}{2}=\frac{50}{8}=\frac{25}{4}$

۱۰

اگر f تابعی خطی باشد، $f(x)+f\left(\frac{1}{x}\right)=\frac{3x^2-12x+3}{x}$ ، مقدار $f(-1)$ برابر با ؟

$ax+b+a\frac{1}{x}+b=a\left(\frac{1}{x}+x\right)+b$

$\frac{3x^2}{x}-\frac{12x}{x}+\frac{3}{x}=3x-12+\frac{3}{x}=3\left(\frac{1}{x}+x\right)-12$

$a=3$

$b=-9$

$f(-1)=3(-1)-9=-9$