

$$f(x) = \begin{cases} (a+1)(x+1) & x > 1 \\ 3a+2x & x \leq 1 \end{cases}$$

$$x=0 \rightarrow f(0) = (a+1)(1)$$

$$x=-1 \rightarrow f(-1) = 3a-1$$

$$f(0) = f(-1) + a$$

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$$(a+1)(1) = 3a-1+a$$

$$1(a+1) = 4a-1$$

$$10a = -1 \rightarrow a = -1, 1 = -\frac{9}{a}$$

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$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 1$$

$$(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y}) = \sqrt{(x-y)(x+y)} = \sqrt{x-y} = 1$$

$$\sqrt{x}+\sqrt{y} = \frac{1}{\sqrt{x}-\sqrt{y}} \rightarrow f(\sqrt{x}-\sqrt{y}) + f(\sqrt{x}+\sqrt{y}) = \left(\sqrt{x}-\sqrt{y} + \frac{1}{\sqrt{x}-\sqrt{y}} + 1\right) + \left(\sqrt{x}+\sqrt{y} + \frac{1}{\sqrt{x}+\sqrt{y}} + 1\right)$$

$$= (\sqrt{x}-\sqrt{y} + \sqrt{x}+\sqrt{y}) + \left(\frac{1}{\sqrt{x}-\sqrt{y}} + \frac{1}{\sqrt{x}+\sqrt{y}}\right) + 2 = \sqrt{x}+\sqrt{y} + \frac{2}{\sqrt{x}+\sqrt{y}} + 2 = 2\sqrt{x} + 2$$

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$$\frac{\sqrt{x}+\sqrt{y}+\sqrt{x}-\sqrt{y}}{\sqrt{(x-y)(x+y)}} = \frac{\sqrt{x}}{\sqrt{x-y}} = \sqrt{y}$$

$$(\sqrt{x}-\sqrt{y} + \sqrt{x}+\sqrt{y})^2 = (x-y) + (x+y) + 2\sqrt{x-y} = 2 + 2 = 4 \rightarrow \sqrt{x}-\sqrt{y} + \sqrt{x}+\sqrt{y} = \sqrt{4}$$

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$$\begin{cases} 2f(x) - 3f(-x) = 4x^2 - 2x \\ 2f(-x) - 3f(x) = -4x^2 + 2x \end{cases}$$

$$\begin{cases} 4f(x) - 9f(-x) = 12x^2 - 2x \\ 4f(-x) - 9f(x) = -12x^2 + 2x \end{cases}$$

$$-2f(x) = -4x^2 + 2x \rightarrow$$

$$f(x) = \frac{-4x^2 + 2x}{-2}$$

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$$(x+1)f(x) - 3xf(x+1) = 4x^2 - mx + m-1$$

$$x=0 \rightarrow 1f(0) - 0 = 4m-1 \rightarrow f(0) = \frac{4m-1}{1}$$

$$x=-1 \rightarrow 0 \cdot f(-1) - (-4)f(0) = 4 + 4m + m-1 \rightarrow 4f(0) = 5 + 5m$$

$$\rightarrow f(0) = \frac{5 + 5m}{4}$$

$$\frac{4m-1}{1} = \frac{5(m+1)}{4} \xrightarrow{\times 4} 4m-1 = 5m+5$$

$$\leftarrow m = \frac{11}{4} = \frac{9}{1}$$

$$f(0) = \frac{4m-1}{1} = \frac{4 \cdot \frac{11}{4} - 1}{1} = \frac{11-1}{1} = \frac{10}{1}$$

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$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 10}{2x}$$

$$\rightarrow ax+b + \frac{a}{x} + b = ax + \frac{a}{x} + 2b$$

$$\frac{3x^2}{2x} - \frac{12x}{2x} + \frac{10}{2x} = 3x - 12 + \frac{10}{2x}$$

$$ax + \frac{a}{x} + 2b = 3x - 12 + \frac{10}{2x}$$

$$a=3$$

$$2b = -12 \rightarrow b = -6$$

↓

$$f(x) = 3x - 6$$

$$f(-1) = -3 - 6 = -9$$

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