

14, 15

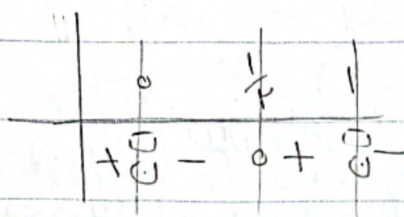
نویسنده ارشد - گروه ریاضیات و فیزیک - تالیف و تصحیح

1) الف) $f(x) = \sqrt{\frac{x-1}{x} - \frac{x}{x-1}} \geq 0$

$$\frac{(x-1)(x-1)}{x(x-1)} - \frac{x(x)}{(x-1)(x)} = \frac{x^2 - 2x + 1 - x^2}{x(x-1)} = \frac{-2x+1}{x(x-1)}$$

$$\frac{-2x+1}{x(x-1)} \geq 0 \quad -2x+1 \geq 0 \quad 2x \leq 1 \quad x \leq \frac{1}{2}$$

$$x(x-1) \neq 0$$



$(-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{\frac{1}{x+1} - \frac{x}{x}}{\frac{x}{x-1} + \frac{1}{x+1}} = \frac{\frac{x - x^2 - x^2}{x^2 + x}}{\frac{x^2 + x + x - 1}{x^2 + x - 1}}$

$$\frac{(-x^2 - x)(x^2 + x - 1)}{(x^2 + x)(x^2 + x)}$$

$x-1 \neq 0 \rightarrow x \neq 1$

$x+1 \neq 0 \rightarrow x \neq -1$

مجموعه حلال = $0, -\frac{1}{x}, -1$ $D_f = \mathbb{R} - \{0, -1, -\frac{1}{x}\}$

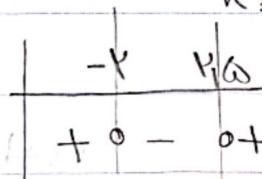
2) الف) $f(x) = \sqrt{\left(\frac{1}{x} - 9\right)(3x - x)} \geq 0$

$$\frac{1}{x} - 9 = 0$$

$$x = -\frac{1}{9}$$

$$3x - x = 0$$

$$x = \frac{1}{2}$$



$(-\infty, -\frac{1}{9}] \cup [\frac{1}{2}, +\infty)$

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ب) $\phi(x) = (\sqrt{x-1} + \sqrt{y+1} = 3)^2$

$\sqrt{x-1} + y+1 = 9$ $\xrightarrow{x-1 \rightarrow x \geq 1}$ $\sqrt{x-1} + \sqrt{y+1} \rightarrow x \leq 12 - D_\phi = [1, 12]$

$y = 9 - 1 - \sqrt{x-1}$

$y = 8 - \sqrt{x-1}$ $\sqrt{x-1} \geq 0$ $x \geq 1$

$D_\phi = [1, +\infty)$

۳) $\phi(x) = \frac{\log(x^2 - x - 1)}{\sqrt{x^2 - 1} + 1}$

$x^2 - x - 1 > 0$
 $(x-1)(x+1) > 0$

$\begin{array}{c|cc} & -1 & 1 \\ \hline & + & - \end{array}$

$\sqrt{x^2 - 1} \geq 0$

$\mathbb{R} - \{0\}$ I

II $(-\infty, -1) \cup (1, +\infty)$

با Silic هر یک می توان نوشت

$D_\phi = I \cap II = (-\infty, -1) \cup (1, +\infty)$

۴) $\phi(x) = \sqrt{3 + ax - x^2} \geq 0$ $[-a, b]$

$3 - 2a - 4 = 0$

$-2a = 1$ $a = -\frac{1}{2}$

$3 - \frac{1}{2}x - x^2 \geq 0$

$2x^2 + x - 6 \geq 0$

$x^2 + x - 3 \geq 0$

$(x+4)(x-3) \geq 0$

$(x+3)(x-4) \geq 0$

$-3 + \frac{3}{2}$

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$\begin{array}{c|cc} & -1 & 1 \\ \hline & - & + \end{array}$

$b = \frac{3}{2}$

$a+b = -\frac{1}{2} + \frac{3}{2} = 1$

$-\frac{1}{2} + \frac{3}{2} = 1$

$$a) \quad f(x) = \begin{cases} 3x-2 : x > 1 \\ 2x+2 : x < 1 \end{cases} \quad -2 \rightarrow \text{مقی}$$

سوال: $g(x) = \sqrt{f(x)} - x \geq 0$

$$f(x) \geq x$$

سریعاً $x = -2, -1, 0, 1, \dots$

$$x = -1$$

$$-1+2 = 1$$

$$D_f = [-2, +\infty)$$

$$c) \quad f(x) = \begin{cases} (a+1)x+2 : x > 1 \\ 3a+2x : x < 1 \end{cases}$$

$$f(a) = f(-1) + a$$

$$f(a) = 1, 100$$

$$f(a) = (a+1)(1) = 1a+1$$

$$1a+1 = 3a-1+a$$

$$f(-1) = 3a-1$$

$$10a = -11$$

$$a = -1, 1$$

$$v) \quad f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2$$

$$(\sqrt{2-\sqrt{2}} + \sqrt{2+\sqrt{2}})^2 = 4$$

$$f(2-\sqrt{2}) + f(2+\sqrt{2}) = ?$$

$$\sqrt{2-\sqrt{2}} + \frac{1}{\sqrt{2-\sqrt{2}}} + 2 + \sqrt{2+\sqrt{2}} + \frac{1}{\sqrt{2+\sqrt{2}}} + 2 = 4 + 2\sqrt{4}$$

مجموع

$$\sqrt{\frac{1}{2-\sqrt{2}} + \frac{1}{2+\sqrt{2}}} + 2 = \sqrt{4}$$

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$$1) \quad 2f(u) - 3f(-u) = 5u^2 - u \times 1^2$$

$$2f(-u) - 3f(u) = 5u^2 + u \times 1^2 \quad (-u) \text{ نوسه}$$

$$2f(u) - 3f(-u) = 11u^2 - 11u$$

$$2f(-u) - 3f(u) = 11u^2 + 11u$$

$$-5f(u) = 10u^2 + 11u$$

$$f(u) = -2u^2 - \frac{11}{5}u$$

$$4) \quad (u+1)f(u) - 3uf(u+1) = 5u^2 - mu + 11m-1$$

$$u = -1$$

$$0 = 14 + 11m + 11m - 1$$

$$0 = 13 + 22m$$

$$m = -\frac{13}{22}$$

$$2f(0) = -9 - 1$$

$$f(0) = -5$$

$$10) \quad f(u) + f\left(\frac{1}{u}\right) = \frac{5u^2 - 11u + 11}{u}$$

$$f(-1) = ?$$

$$y = 5u - 4$$

$$f(-1) = y = -5 - 4 = -9$$

$$f(u) = au + b$$

$$f\left(\frac{1}{u}\right) = \frac{a}{u} + b$$

$$f(u) + f\left(\frac{1}{u}\right) = au + \frac{a}{u} + b$$

$$\frac{5u^2 - 11u + 11}{u} = 5u - 11 + \frac{11}{u}$$

$$a = 5$$

$$b = -4$$

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