

ب) $\frac{1}{n} - \frac{1}{n-1}$

٢٥

٢٥

٢٥

$$\text{ا) } \sqrt{\frac{n-1}{n} - \frac{n}{n-1}} \Rightarrow n \neq 1, 0 \quad \frac{n-1}{n} - \frac{n}{n-1} > 0 \quad \frac{n^2 - 2n + 1 - n^2}{n^2 - n} \geq 0 \quad (1)$$

$$(1), (2) \Rightarrow D_f = (-\infty, 0) \cup \left[\frac{1}{r}, 1\right)$$

٥

$$+ \frac{1}{0} - \frac{1}{0} + \frac{1}{0} -$$

$$\text{ب) } \frac{\frac{1}{n+1} - \frac{1}{n}}{\frac{1}{n-1} + \frac{1}{n+r}} \Rightarrow n \neq -1, -1, 0, 1 \quad \frac{1}{n-1} + \frac{1}{n+r} \neq 0$$

(1)

$$\frac{r(n+1)}{(n-1)(n+r)} \neq 0$$

$$\frac{r(n+1) + n-1}{(n-1)(n+r)} \neq 0$$

$$(2) \quad n \neq -1, -\frac{1}{r}, 1$$

$$(1), (2) \Rightarrow D_f = \mathbb{R} \setminus \left\{-1, -\frac{1}{r}, 1\right\}$$

$$\text{ا) } \left(\left(\frac{1}{r}\right)^n - 9\right) (r^2 - \varepsilon^n) \geq 0 \quad \frac{-r}{+0} - \frac{\frac{1}{r}}{+} \quad D_f = (-\infty, -r] \cup \left[\frac{1}{r}, +\infty\right) \quad (2)$$

$$\log_{\frac{1}{r}} 9 = -r \quad \log_{\varepsilon} r^2 = \frac{2}{r}$$

٥

$$\text{ب) } \sqrt[n]{n+1} + \sqrt[n]{y+1} \leq r \rightarrow \sqrt[n]{y+1} \leq r - \sqrt[n]{n+1} \rightarrow n-1 \geq 0 \quad n \geq 1 \quad (1)$$

$$r - \sqrt[n]{n+1} \rightarrow \sqrt[n]{n+1} \leq r \quad n-1 \leq 1 \quad n \leq 2 \quad D_f = [1, 2]$$

$$\frac{\log_2 (n^2 - n - 2)}{\sqrt{n^2 - 1} + 1} \rightarrow \log_2 (n^2 - n - 2) \rightarrow n^2 - n - 2 > 0 \quad \frac{1}{+0} - \frac{r}{+} \quad (-\infty, -1) \cup (r, +\infty) \quad (2)$$

٥

(1)

$$D_f = (-\infty, -1) \cup (r, +\infty)$$

(2)

(1), (2)

$$n = -1 \quad r - ra - \varepsilon = 0 \quad ra = -1 \quad a = -\frac{1}{r} \rightarrow -n^2 - \frac{1}{r}n + r \xrightarrow{\text{discriminant } \Delta} \quad \textcircled{\varepsilon}$$

$$\Delta = \frac{1}{\varepsilon} - r(-r) = \frac{r^2}{\varepsilon} \quad n = \frac{-r \pm \sqrt{\frac{r^2}{\varepsilon}}}{2} = \frac{-r \pm r}{2} \quad \textcircled{\varepsilon} \quad \boxed{a+b=1}$$

$$g(n) = \sqrt{f(n) - n}$$

$$f(n) - n \geq 0$$

$$f(n) \geq n \rightarrow \text{for } n \geq 1 \rightarrow \begin{cases} n-1 \geq n & n-1 \geq 0 \\ n+1 \geq n & n+1 \geq 0 \end{cases} \quad \textcircled{\varepsilon} \quad \textcircled{1}$$

$$\textcircled{1} \textcircled{2} \rightarrow Df[-1, +\infty)$$

$$n+1 \geq n \quad n+1 \geq 0 \quad n \geq -1 \quad \textcircled{2}$$

$$f(n) = \begin{cases} (n+1)(n-1) & n \geq 1 \\ n+1 & n \leq 1 \end{cases}$$

$$rf(a) = f(-r) + a$$

$$\textcircled{\varepsilon} \quad f(a) = (a+1)(v) = va + v$$

$$rf(a) = 1/a + 1/\varepsilon$$

$$f(-r) = ra - \varepsilon$$

$$1/a + 1/\varepsilon = ra - \varepsilon + a$$

$$1/a = -1/\varepsilon$$

$$\boxed{a = -1/\varepsilon}$$

$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + r \rightarrow f(r-\sqrt{r}) + \underbrace{f(r+\sqrt{r})}_{+} \rightarrow f(r+\sqrt{r}) + f\left(\frac{1}{r+\sqrt{r}}\right) \textcircled{V}$$

$$\frac{1}{r+\sqrt{r}} = r-\sqrt{r} \rightarrow f(1) + f\left(\frac{1}{r}\right) = r\left(\sqrt{r} + \frac{1}{\sqrt{r}}\right) + \varepsilon \rightarrow r\left(\underbrace{\sqrt{r+\sqrt{r}} + \frac{1}{\sqrt{r+\sqrt{r}}}}_m\right) + \varepsilon$$

$$m^2 = (r-\sqrt{r})(r+\sqrt{r}) + r\sqrt{(r+\sqrt{r})(r-\sqrt{r})} \rightarrow m^2 = r$$

$$f(r+\sqrt{r}) + f(r-\sqrt{r}) = rm + \varepsilon = r\sqrt{r} + \varepsilon$$

$$rf(n) - rf(-n) = f(n^2 - n) \times r$$

$$rf(-n) - rf(n) = f(n^2 + n) \times r$$

$$rf(-n) - rf(n) = 1f(n^2 + n)$$

$$rf(n) - rf(-n) = 1f(n^2 - n) \textcircled{1}$$

$$\textcircled{3} \rightarrow f(n) = r_0 n^2 + n$$

$$\boxed{f(n) = \frac{r_0 n^2 + n}{-2}}$$

$$(n+r)f(n) - rf(n+r) = f(n^2 - mn + r^2 m - 1)$$

$$rf(0) = r^2 m - 1 \rightarrow n=1$$

$$rf(0) = 2m + 1$$

$$rf(0) = 1r + r^2 m + r^2 m - 1 \rightarrow m = -\textcircled{3}$$

$$-rf(0) = -2m + 1$$

$$rm = 1 \quad m = \frac{1}{r}$$

$$rf(0) = r \times \frac{1}{r} - 1 = \frac{r^2}{r} \quad \boxed{f(0) = \frac{r^2}{2}}$$

$$f(n) + f\left(\frac{1}{n}\right) = \frac{r n^2 - 1 r n + r^2}{n}$$

$$\xrightarrow{n=-1}$$

$$f(-1) + f(-1) = \frac{r+1r+r^2}{-1} \textcircled{10}$$

$$rf(-1) = -1$$

$$\boxed{f(-1) = -\frac{1}{r}}$$

$$\textcircled{3}$$