فاطمه فروتن — تکلیف ۴ — یازدهم دفتر B

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الف) $\sqrt{\frac{x-1}{x}-\frac{x}{x-1}} \Rightarrow x \neq 1, 0$ (1)

$\frac{x-1}{x}-\frac{x}{x-1} \geq 0 \quad \frac{x^2-2x+1-x^2}{x^2-x} \geq 0$

x		0		$\frac{1}{2}$		1	
	$+$	∄	$-$	0	$+$	∄	$-$

(1),(2) $\Rightarrow D_f=(-\infty,0)\cup[\frac{1}{2},1)$

ب) $\dfrac{\frac{1}{x+1}-\frac{2}{x}}{\frac{3}{x-1}+\frac{1}{x+2}} \Rightarrow x \neq -2,-1,0,1$ (1) ، $\frac{3}{x-1}+\frac{1}{x+2} \neq 0$

$\frac{3x+6+x-1}{(x-1)(x+2)} \neq 0 \quad \frac{4x+5}{(x-1)(x+2)} \neq 0$

(2) $x \neq -2,-\frac{5}{4},1$

(1),(2) $\Rightarrow D_f=\mathbb{R}-\{-2,-\frac{5}{4},-1,0,1\}$

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الف) $\sqrt{\left((\frac{1}{3})^x-9\right)\left(32-4^x\right)} \geq 0$

x		-2		$\frac{5}{2}$	
	$+$	0	$-$	0	$+$

$D_f=(-\infty,-2]\cup[\frac{5}{2},+\infty)$

$\log_{\frac{1}{3}}9=-2 \quad \log_4 32=\frac{5}{2}$

ب) $\sqrt[4]{x-1}+\sqrt{y+1}=3 \to \sqrt{y+1}=3-\sqrt[4]{x-1} \to x-1 \geq 0 \quad x \geq 1$ (1)

$3-\sqrt[4]{x-1} \to \sqrt[4]{x-1} \leq 3 \quad x-1 \leq 81 \quad x \leq 82 \quad D_f=[1,82]$

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$\dfrac{\log_4(x^2-x-2)}{\sqrt{x^2-1}+1} \to \log_4(x^2-x-2) \to x^2-x-2>0$

x		-1		2	
	$+$	0	$-$	0	$+$

$(-\infty,-1)\cup(2,+\infty)$ (1)

همواره مثبت $\to x^2-1 \geq 0 \quad x^2 \geq 1 \to x \geq 1 \quad x \leq -1$ (2)

(1),(2) $\Rightarrow D_f=(-\infty,-1)\cup(2,+\infty)$

$x = -2 \quad 3 - 2a - 4 = 0 \quad 2a = -1 \quad a = -\frac{1}{2} \rightarrow -x^2 - \frac{1}{2}x + 3$ Δ به دست آوریم

④

$\Delta = \frac{1}{4} - 4(-3) = \frac{49}{4} \quad x \begin{cases} -2 \\ \boxed{\frac{3}{2} = b} \end{cases} \quad \boxed{a + b = 1}$

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⑤

$f(x) \begin{cases} 3x - 2 & x \geq 1 \\ 2x + 3 & x < 1 \end{cases}$

$g(x) = \sqrt{f(x) - x}$

$f(x) - x \geq 0$

$f(x) \geq x \rightarrow$ برقرار برای $x \geq 1$ $\rightarrow 3x - 2 \geq x \quad 2x - 2 \geq 0$

$x \geq 1$ ①

$2x + 3 \geq x \quad x + 3 \geq 0 \quad x \geq -3$ ②

①,② $\rightarrow D_f \, [-3, +\infty)$

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⑥

$f(x) = \begin{cases} (a+1)(x+2) & x > 1 \\ 3a + 2x & x \leq 1 \end{cases}$

$2f(5) = f(-2) + a$

$f(5) = (a+1)(7) = 7a + 7$

$2f(5) = 14a + 14$

$f(-2) = 3a - 4$

$14a + 14 = 3a - 4 + a$

$10a = -18$

$\boxed{a = -1.8}$

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}} + 2 \rightarrow f(2-\sqrt{3}) + \underbrace{f(2+\sqrt{3})}_{+} \rightarrow f(2+\sqrt{3}) + f\left(\frac{1}{2+\sqrt{3}}\right)$$ 7

$$\frac{1}{2+\sqrt{3}} = 2-\sqrt{3} \rightarrow f(t) + f\left(\frac{1}{t}\right) = 2\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right) + 4 \rightarrow 2\underbrace{\left(\sqrt{2+\sqrt{3}} + \frac{1}{\sqrt{2+\sqrt{3}}}\right)}_{m} + 4$$

$$m^2 = (2-\sqrt{3})(2+\sqrt{3}) + 2\sqrt{(2+\sqrt{3})(2-\sqrt{3})} \rightarrow m^2 = 6$$

$$f(2+\sqrt{3}) + f(2-\sqrt{3}) = 2m + 4 = 2\sqrt{6} + 4$$

$$2f(x) - 3f(-x) = 4x^2 - x \quad \times 3$$

$$2f(-x) - 3f(x) = 4x^2 + x \quad \times 2$$

$$6f(-x) - 9f(x) = 12x^2 + 3x$$

$$4f(x) - 6f(-x) = 8x^2 - 2x$$ 8

$$-5f(x) = 20x^2 + x$$

$$\boxed{f(x) = \frac{20x^2 + x}{-5}}$$

$$(x+2)f(x) - 2xf(x+2) = 4x^2 - mx + 3m - 1$$

$$2f(0) = 3m - 1 \rightarrow x = 1$$ 9

$$6f(0) = 16 + 2m + 3m - 1 \rightarrow x = -2$$

$$6f(0) = 5m + 15$$

$$-6f(0) = -9m + 3$$

$$4m = 18 \qquad m = \frac{9}{2}$$

$$2f(0) = 3 \times \frac{9}{2} - 1 = \frac{25}{2} \qquad \boxed{f(0) = \frac{25}{4}}$$

$$f(x) + f\left(\frac{1}{x}\right) = \frac{3x^2 - 12x + 3}{x} \xrightarrow{x=-1} f(-1) + f(-1) = \frac{3+12+3}{-1}$$ 10

$$2f(-1) = -18 \qquad \boxed{f(-1) = -9}$$