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نام و نام خانوادگی پاسخنامه تشریحی تکلیف شماره کلاس نام و نام خانوادگی الینا علی نجاری

الف) $f(x) = \sqrt{\frac{x-3}{x}} - \frac{x}{x-1} \rightarrow \sqrt{\frac{(x-1)^2 - x^2}{x(x-1)}} = \sqrt{\frac{x^2+1-2x-x^2}{x(x-1)}} = \sqrt{\frac{-2x+1}{x(x-1)}} \rightarrow \frac{0}{+0} - \frac{1}{0} \rightarrow \frac{0}{+0} - \frac{1}{0}$

$Df = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $f(x) = \frac{1}{x+1} + \frac{1}{x} \rightarrow x+1 \neq 0 \rightarrow x \neq -1, x \neq 0$
 $\frac{x}{x-1} + \frac{1}{x+2} \rightarrow x-1 \neq 0 \rightarrow x \neq 1, x+2 \neq 0 \rightarrow x \neq -2, \frac{x}{x-1} + \frac{1}{x+2} \neq 0 \rightarrow \frac{x(x+2) + (x-1)}{(x-1)(x+2)} \neq 0 \rightarrow \frac{x^2+3x-1}{(x-1)(x+2)} \neq 0$

$Df = \mathbb{R} - \{-2, -\frac{1}{3}, \frac{1}{2}, 1, 0\}$

الف) $f(x) = \sqrt{\left(\left(\frac{1}{x}\right)^2 - 9\right)} \left(4x - \frac{x}{x-1}\right), 4x - \frac{x}{x-1} \geq 0 \rightarrow \frac{4x(x-1) - x}{x(x-1)} \geq 0 \rightarrow \frac{4x^2 - 5x}{x(x-1)} \geq 0 \rightarrow \frac{x(4x-5)}{x(x-1)} \geq 0 \rightarrow \frac{4x-5}{x-1} \geq 0$
 $\frac{4x-5}{x-1} \geq 0 \rightarrow \frac{-2}{+0} \rightarrow x > \frac{5}{4}, x < 1$
 $Df = (-\infty, -2] \cup [\frac{5}{4}, +\infty)$

ب) $\sqrt{x-1} + \sqrt{y+1} \geq \sqrt{y+1} \rightarrow \sqrt{x-1} \geq 0 \rightarrow x-1 \geq 0 \rightarrow x \geq 1$
 $\sqrt{x-1} \leq \sqrt{y+1} \rightarrow x-1 \leq y+1 \rightarrow x \leq y+2$
 $Df = [1, 14]$

$f(x) = \frac{\log(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \rightarrow \frac{-1}{+0} \rightarrow x > 2, x < -1$
 $\frac{1}{(x-1)(x+1)} > 0 \rightarrow \frac{1}{+0} \rightarrow x > 1, x < -1$
 $\frac{1}{(x-1)(x+1)} > 0 \rightarrow \frac{1}{+0} \rightarrow x > 1, x < -1$
 $\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1 \rightarrow x^2 - 1 \neq 1 \rightarrow x^2 \neq 2 \rightarrow x \neq \pm\sqrt{2}$
 $Df = (-\infty, -1) \cap (2, +\infty)$

$a+b \in [a, b] \iff f(x) \in [a, b] \iff \sqrt{r_1 a n - n^2} \in [a, b]$
 $-n^2 + a n + r_1 = (n+r)(n-b) \rightarrow n^2 - a n - r_1 = (n+r)(n-b)$
 $n^2 - a n - r_1 = (n+r)(n-b) \rightarrow -r_1 b = -r_1 \rightarrow b = \frac{r_1}{r_1} = 1$
 $n^2 - a n - r_1 = n^2 + \frac{1}{r_1} n - r_1 \rightarrow a = -\frac{1}{r_1} \rightarrow a+b = \frac{r_1}{r_1} - \frac{1}{r_1} = 1$

$f(x) = \begin{cases} x^{n-r}, & x \geq 1 \\ x^{n+r}, & x < 1 \end{cases} \rightarrow \begin{cases} x^{n-r} \geq x \\ x^{n+r} \geq x \end{cases}$
 $x^{n+r} \geq x \rightarrow x^{n+r-1} \geq 1 \rightarrow x \geq -1 \cap x < 1 \rightarrow -1 \leq x < 1$
 $Df = [-1, +\infty)$

$$f(n) = \begin{cases} (a+1)(n+1) & ; n > 1 \\ 3a+1 & ; n \leq 1 \end{cases}, \quad \text{if } f(a) = f(-1) + a \rightarrow a ?$$

$$f(a) = (a+1)(1) = 2a+1, \quad f(-1) = 3a - 2 \quad \left\{ \begin{array}{l} 1 \leq a+1 \leq 3a-2+a \\ 1 \cdot a > -1 \rightarrow a > -1, \wedge \end{array} \right.$$

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$$f(n) = \sqrt{n} + \frac{1}{\sqrt{n}} + 1 \rightarrow f(\sqrt{n}) + f(\sqrt{n}) ?$$

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$$\sqrt{n} = x_1, \quad \frac{1}{\sqrt{n}} = x_2, \quad \sqrt{n} = x_1, \quad \frac{1}{\sqrt{n}} = x_2$$

$$f(n_1) = \sqrt{n_1} + \frac{1}{\sqrt{n_1}} + 1 + \sqrt{n_2} + \frac{1}{\sqrt{n_2}} + 1 = 2 + \sqrt{n_1} + \sqrt{n_2} + \frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}} = 2 + \sqrt{\sqrt{n_1} + \sqrt{n_2}} + \sqrt{\frac{1}{\sqrt{n_1}} + \frac{1}{\sqrt{n_2}}}$$

$$f(n) - f(-n) = 2n^2 - n$$

$$n \geq 0 \rightarrow f(n) - f(-n) = 2n^2 - n \xrightarrow{\times 1} -4f(-n) + f(n) = 12n^2 - 2n$$

$$n \geq -n \rightarrow f(-n) - f(n) = 2n^2 + n \xrightarrow{\times 1} 4f(-n) - f(n) = 12n^2 + 2n$$

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$$-af(n) = 2n^2 + n$$

$$f(n) = 2n^2 - \frac{n}{2}$$

$$(n+1)f(n) - n f(n+1) = 2n^2 - mn + m - 1$$

$$n \geq 0 \rightarrow f(0) = 1 + m - 1 \xrightarrow{\times \omega} f(0) = 10m - 1$$

$$n \geq -1 \rightarrow 4f(0) = 14 + m + m - 1 = 13 + 2m \xrightarrow{\times 1} 14f(0) = 14m + 14$$

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$$14f(0) = 14$$

$$f(0) = \frac{14}{14}$$

$$f(n) + f\left(\frac{1}{n}\right) = \frac{1}{n} + 1 + \frac{1}{n} \rightarrow f(n) = an + b$$

$$n \geq -1 \rightarrow f(-1) = \frac{1 + 1 + 1}{-1} = -3 \rightarrow f(-1) = -3$$

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