

۱۹,۵

$$\sqrt{x^r \left( \left( \frac{1}{r} \right)^{x-r+1} - 1 \right)} = \sqrt{\frac{x^r}{r^{x-1}} - x^r} \rightarrow \frac{x^r}{r^{x-1}} x^r \rightarrow x^r (r^{-x+1}) - x^r$$

$$\frac{0-x}{x^r} (r^{-x+1} - 1) \geq 0 \rightarrow \frac{x}{-x+1} \rightarrow D_f = [0, 1]$$

$$r^{-x+1} = 1 \rightarrow 1-x=0 \rightarrow x=1$$

$$f(-x) = \sqrt{-x + |-x+2|} = \sqrt{-x + |x-2|} \rightarrow -x + |x-2| \geq 0 \rightarrow |x-2| \geq x$$

$$\begin{cases} x-2 \geq x \rightarrow -2 \geq 0 \times \\ x-2 \leq -x \rightarrow 2x \leq 2 \rightarrow x \leq 1 \end{cases} \Rightarrow D_f = (-\infty, 1]$$

$$\frac{x-1}{f(x)} \geq 0 \rightarrow \frac{x}{-1 + \frac{1}{r} + \frac{r}{x}} \Rightarrow D_f = (-\varepsilon, 1] \cup (r, r)$$

با توجه به نمودار  $\rightarrow +^r, r, -\varepsilon$

$$9x^2 - 16x^r - \varepsilon x - r = (3x^r - x)(3x^r + x) - 4x^r - \varepsilon x - r$$

$$(3x^r - x + \omega)(3x^r + x + \omega) = 9x^2 + x^r(3\omega - 1 + 3\omega) + x(-2 + \omega) + \omega^2$$

$$\begin{cases} 3\omega - 1 + 3\omega = -\varepsilon \\ -2 + \omega = -\varepsilon \end{cases} \rightarrow \begin{cases} 6\omega - 1 = -\varepsilon \\ \omega = 1 - \varepsilon \end{cases} \rightarrow \begin{cases} 6(1 - \varepsilon) - 1 = -\varepsilon \\ 6 - 6\varepsilon - 1 = -\varepsilon \\ 5 - 6\varepsilon = -\varepsilon \\ 5 = 5\varepsilon \\ \varepsilon = 1 \end{cases}$$

$$\omega = 1 - \varepsilon = 0 \rightarrow \omega = 0$$

$$(3x^r - x)(3x^r + x) = 9x^2 - x^2 = 8x^2$$

$$\sqrt{8x^2} = 2\sqrt{2}x$$

$$y = \sqrt{x^r + x - \frac{4\varepsilon}{x^r} - \frac{\varepsilon}{x}} \rightarrow \frac{x^r + x - 4\varepsilon - \varepsilon x^r}{x^r} \rightarrow \begin{cases} x^r = t \rightarrow t^r + t - \varepsilon t - 4\varepsilon \\ t^r - 4\varepsilon + t(t - \varepsilon) \rightarrow (t - \varepsilon)(t^r + \varepsilon t + 4) + t(t - \varepsilon) \\ (t - \varepsilon)(t^r + \varepsilon t + 4) \end{cases}$$

$$t = x^r = \varepsilon \rightarrow x = \varepsilon$$

$$D_f = [5, 0) \cup [r, +\infty)$$

$$f(r) = f(r+1) = r-1 = \boxed{1}$$

$$f_{(10)} = f(r+r) = r-1 = \boxed{v}$$

$$\Rightarrow f(r) + f_{(10)} = 1+v = \boxed{r}$$

$$f(u) = (u-r)^r + r \rightarrow \frac{g(\sqrt{v}-r)}{f(\sqrt{r}+r)} = \frac{(\sqrt{v}+r-r)^r - rv}{(\sqrt{r}+r-r)^r + r}$$

$$g(u) = (u+r)^r - rv$$

$$\Rightarrow \frac{v-rv}{r+r} = \frac{-r_0}{1_0} = \boxed{-r}$$

$$f(u) + g(u) = \sqrt{(u-r) - \varepsilon \sqrt{u-r} + \varepsilon} + \sqrt{(u-r) + \varepsilon \sqrt{u-r} + \varepsilon} = \sqrt{(\sqrt{u-r} + r)^2}$$

$$\sqrt{(\sqrt{u-r} - r)^2} + \sqrt{(\sqrt{u-r} + r)^2} = |\sqrt{u-r} - r| + |\sqrt{u-r} + r| \xrightarrow{\substack{u > r \\ \varepsilon > 0}} |\sqrt{u-r} + r|$$

$$\begin{cases} \sqrt{u-r} + r - \sqrt{u-r} + r = \varepsilon ; \quad u > r, \varepsilon \end{cases}$$

$$\begin{cases} \sqrt{u-r} + r + \sqrt{u-r} - r = 2\sqrt{u-r} ; \quad u < r \Rightarrow \begin{cases} a=0 \\ b=r \\ c=-\varepsilon \end{cases} \Rightarrow \frac{a+b}{c} = \frac{r}{-\varepsilon} = \boxed{-\frac{1}{\varepsilon}}$$

$$f(u) = \left\{ \left( r, \frac{\varepsilon}{\varepsilon-r+r} \right), \left( 0, \frac{r}{r} \right) \right\} \rightarrow f(u) = \frac{u+r}{u^2 - \varepsilon u + r} \rightarrow D_f = \mathbb{R} - \{1, r\}$$

$$\frac{g}{f} = \left\{ \left( r, -\frac{1}{\varepsilon} \right), \left( 0, r \right) \right\}$$

$$الف) \left\{ \left( \frac{1}{r}, r \right), \left( \frac{r}{r}, -1 \right), \left( r, r \right), \left( -\frac{1}{r}, r \right) \right\}$$

$$ب) \left\{ \left( 1, r \right), \left( \sqrt{r}, -1 \right), \left( r, r \right) \right\}$$

$$ج) \left\{ \left( -r, 1/r \right), \left( 1, r \right), \left( r, r \right), \left( -1, 1 \right) \right\}$$

$$د) \left\{ \left( -1, \frac{1}{r} \right), \left( r, -1 \right), \left( 1, -\varepsilon \right) \right\}$$

$$\hookrightarrow \left\{ \left( r, -1 \right), \left( 1, -\varepsilon \right) \right\}$$

$$\boxed{1, r}$$

$$ب. 10) \quad \begin{cases} x^r = 1 \rightarrow x = \pm 1 \\ x^r = r \rightarrow x = \pm r \\ x^r = -1 \rightarrow x = \pm i \end{cases}$$

$$\rightarrow \left\{ (\pm 1, r), (\pm \sqrt{r}, -1), (\pm r, r) \right\}$$