سارینا جعفری

تکلیف ۵

۱

$f(x) = (\frac{1}{2})^{x-2} - 1 \qquad y = \sqrt{x^3 f(x+1)} \qquad D_y = ?$

$\to f(x+1) = (\frac{1}{2})^{x-1} + 1 = (2)^{1-x} - 1 \qquad \underbrace{x^3}_{0} \times (2^{1-x} - 1) \geq 0$

$\to 1-x=0 \Rightarrow x=1$

$$\begin{array}{c|c|c} & 0 & 1 & \\ - & + & - \end{array}$$

x^3 منفی $\quad \to 2^{1-x}-1$ منفی

$D_y = [0, 1]$

۲

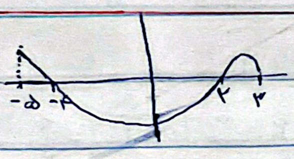
$f(x) = \sqrt{x + |x+2|} \qquad f(-x) \to D = ?$

$\to f(-x) = \sqrt{-x + |2-x|} \Rightarrow -x + |2-x| \geq 0 \qquad |2-x| \geq x$

$2-x \geq x \to 2x \leq 2 \to x \leq 1 \qquad D_f = (-\infty, 1]$

$2-x \leq -x \qquad 2 \leq 0$ غلط

۳



$y = \sqrt{\frac{x-1}{f(x)}} \qquad \frac{x-1}{f(x)} \geq 0$

$$\begin{array}{ccccccccc} & -4 & & 1 & & 2 & & 3 & \\ - & \nexists & + & 0 & - & \nexists & + & \nexists & - \end{array}$$

$x-1=0 \to x=1$

$f(x)=0 \to x = 3, 2, -4 \qquad D_f = (-4, 1] \cup (2, 3)$

۴

$y_1 = \frac{1}{9x^4 - 7x^2 - 4x - 3} \qquad y_2 = \frac{1}{3x^2 + ax + b} \qquad D_{y_1} = D_{y_2}$

$\sqrt{-(a+b)} = ?$

$9x^4 - 7x^2 - 4x - 3 = 9x^4 - 6x^2 + 1 - (x^2 + 4x + 4) = (3x^2-1)^2 - (x+2)^2 \neq 0$

$3x^2 - 1 = x + 2 \qquad 3x^2 - x - 3 = 0 \xrightarrow{\text{ریشه‌ها برابر است}} 3x^2 - x - 3 = 3x^2 + ax + b$

$a = -1$

$b = -3$

$\sqrt{-(\underbrace{-1-3}_{-4})} = \sqrt{4} = \pm 2$

۵

$f(x) = x^3 + x \qquad y = \sqrt{f(x) - f(\frac{4}{x})} \qquad D_y = ?$

$x^3 + x - (\frac{4}{x})^3 - \frac{4}{x} \geq 0 \qquad x^3 + x \geq \frac{64}{x^3} + \frac{4}{x} \qquad x^3 - \frac{64}{x^3} \geq \frac{4}{x} - x$

$(x - \frac{4}{x})(x^2 + \frac{16}{x^2} + 4) + x - \frac{4}{x} \geq 0 \qquad \underbrace{(x - \frac{4}{x})}_{x = \pm 2}(x^2 + \frac{16}{x^2} + 4 + 1) \geq 0$

$\to \frac{(x^4 + 16 + 5x^2)}{x^2} \to 25 - 4(16) \to \Delta < 0$ همواره مثبت

$x^2 \to x \neq 0$

$$\begin{array}{ccccccc} & -2 & & 0 & & 2 & \\ - & 0 & + & \nexists & - & 0 & + \end{array}$$

$D_y = [-2, 0) \cup [2, +\infty)$

۶

$f(x^3+x)=2x^2-1 \qquad f(2)+f(10)=?$

$x^3+x=2 \rightarrow (1)^3+1=2 \Rightarrow x=1$

$x^3+x=10 \rightarrow (2)^3+2=10 \Rightarrow x=2$

$$f(2)+f(10)=\underset{2}{\cancel{2(1)}}-1+\underset{8}{\cancel{2(4)}}-1=1+7=8$$

۷

$f(x)=x^3-6x^2+12x \qquad g(x)=x^3+9x^2+27x \qquad \dfrac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}$

$$f(\sqrt[3]{2}+2)=(2+8+\cancel{6\sqrt[3]{4}}+\cancel{12\sqrt[3]{2}})-6(\cancel{\sqrt[3]{4}}+\cancel{4\sqrt[3]{2}}+4)+12(\cancel{\sqrt[3]{2}}+2)=10-\cancel{24}+\cancel{24}=10$$

$$g(\sqrt[3]{7}-3)=(\underbrace{7-27}_{-20}-\cancel{9\sqrt[3]{49}}+\cancel{27\sqrt[3]{7}})+9(\cancel{\sqrt[3]{49}}-\cancel{6\sqrt[3]{7}}+9)+27(\cancel{\sqrt[3]{7}}-3)=-20+81-81=-20$$

$$\frac{g(\sqrt[3]{7}-3)}{f(\sqrt[3]{2}+2)}=\frac{-20}{10}=-2$$

۸

$f(x)=\sqrt{x+4\sqrt{x-4}} \qquad g(x)=\sqrt{x-4\sqrt{x-4}} \qquad x\geq 4 \Rightarrow f(x)+g(x)=a+b\sqrt{x+c}$

$\dfrac{a+b}{c}=?$

$$f(x)=\sqrt{(x-4)+4\sqrt{x-4}+4} \longrightarrow f(x)=\sqrt{(\sqrt{x-4}+2)^2} \Rightarrow f(x)=|\sqrt{x-4}+2|$$

$$g(x)=\sqrt{(x-4)-4\sqrt{x-4}+4} \Rightarrow g(x)=|\sqrt{x-4}-2| \xrightarrow[f(x)+g(x)=a+b\sqrt{x+c}]{4\leq x<8} \sqrt{x-4}+2+2-\sqrt{x-4}=4$$

$$\xrightarrow{x\geq 8} \sqrt{x-4}+2+\sqrt{x-4}-2=0+2\sqrt{x-4} \qquad 0+2\sqrt{x-4}=a+b\sqrt{x+c}$$

$a=0,\ b=2,\ c=-4$

$$\frac{a+b}{c}=\frac{0+2}{-4}=-\frac{1}{2}$$

۹

$f(x)=\dfrac{x+2}{x^2-4x+3} \qquad g(x)=\{(2,1)(1,0)(3,5)(0,4)\} \qquad \dfrac{g}{f}$

$$\frac{g}{f}=\left\{\left(2,\frac{1}{-4}\right)\cancel{\left(1,\frac{0}{\frac{3}{0}}\right)}\cancel{\left(3,\frac{5}{\frac{5}{0}}\right)}\left(0,\frac{4}{\frac{2}{3}}\right)\right\} \Rightarrow g/f=\left\{\left(2,-\frac{1}{4}\right)(0,6)\right\}$$

$f(2)=\dfrac{4}{4-8+3}=-4 \qquad f(3)=\dfrac{5}{9-12+3}=\dfrac{5}{0}$ ت ن

$f(1)=\dfrac{3}{1-4+3}=\dfrac{3}{0}$ ت ن $\qquad f(0)=\dfrac{2}{3}$

۱۰

$f(x)=\{(1,2)(3,-1)(4,2)(-1,6)\} \qquad g(x)=\{(-2,3)(1,-1)(3,2)(-1,0)\}$

الف) $f(2x)=\left\{\left(\frac{1}{2},2\right)\left(\frac{3}{2},-1\right)(2,2)\left(\frac{-1}{2},6\right)\right\}$

ب) $f(x^2)=\{(1,2)(\sqrt{3},-1)(2,2)\}$

ج) $2g^2(x)+1=\{(-2,19)(1,3)(3,9)(-1,1)\}$

د) $\dfrac{2f}{g}=\left\{\left(1,\frac{4}{-1}\right)\left(3,\frac{-2}{2}\right)\cancel{\left(-1,\frac{12}{0}\right)}\right\} \Rightarrow \{(1,-4)(3,-1)\}$

مشترک‌ها

$\rightarrow x=1,\ 3,\ -1$