

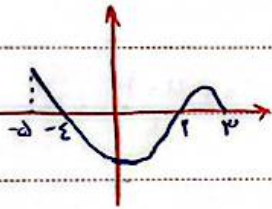
المسألة الأولى

بإيجاد دمجها

المسألة الأولى

$$\begin{aligned} x^r f(n+1) > 0 &\rightarrow \text{if } n > 0 : f(n+1) > 0 \Rightarrow \left(\frac{1}{y}\right)^{n+1-r} - 1 > 0 \Rightarrow y^{1-n} > 1 \\ &\Rightarrow 1-n > 0 \Rightarrow n < 1 \Rightarrow [0, 1] \\ &\rightarrow \text{if } n < 0 : f(n+1) < 0 \Rightarrow \left(\frac{1}{y}\right)^{n+1-r} - 1 < 0 \Rightarrow y^{1-n} < 1 \\ &\Rightarrow 1-n < 0 \Rightarrow n > 1 \quad \times \Rightarrow D = [0, 1] \end{aligned}$$

$$f(-n) = \sqrt{1-n} - n \rightarrow \frac{1-n}{1-n} \geq 0 \Rightarrow 1-n \geq 0 \Rightarrow n \leq 1 \Rightarrow D = (-\infty, 1]$$



$$\begin{aligned} 1) & \quad 1 < n < 3 \quad n-1 > 0 \rightarrow \frac{n-1}{f(n)} > 0 \quad \checkmark \\ 2) & \quad 1 < n < 2 \quad n-1 > 0 \rightarrow \frac{n-1}{f(n)} < 0 \quad \times \quad \Delta) f(n) \neq 0 \\ 3) & \quad -2 < n < 1 \quad n-1 < 0 \rightarrow \frac{n-1}{f(n)} > 0 \quad \checkmark \\ 4) & \quad -2 < n < -1 \quad n-1 < 0 \rightarrow \frac{n-1}{f(n)} < 0 \quad \times \\ \Rightarrow D & = (-2, 1) \cup (1, 3) \end{aligned}$$

$$\begin{aligned} 9n^2 - 4n^2 - 4n - 3 &\Rightarrow (9n^2 - 4n^2 + 1) - (n^2 + 4n + 3) = (5n^2 - 1) - (n+2)^2 \\ &\Rightarrow (5n^2 + 1 + n)(5n^2 - n - 3) \neq 0 \Rightarrow \frac{1}{5n^2 - n - 3} = \frac{1}{5n^2 + an + b} \quad a = -1, b = -3 \\ \Delta < 0 \quad \checkmark \quad \Delta = 1 \\ \sqrt{-(-3)} &= \sqrt{3} \end{aligned}$$

$$\begin{aligned} F(n) - f\left(\frac{\varepsilon}{n}\right) > 0 &\Rightarrow f(n) > f\left(\frac{\varepsilon}{n}\right) \rightarrow n^4 + n > \frac{4\varepsilon}{n^2} + \frac{\varepsilon}{n} \Rightarrow n^4 + n > \frac{4\varepsilon + \varepsilon n^2}{n^2} \\ &\Rightarrow \frac{n^4 + n^2 - 4\varepsilon - \varepsilon n^2}{n^2} > 0 \rightarrow \frac{(n-2)(n+2)(n^2 + 2n + 4)}{n^2} \\ \Delta & = 4 - 16 = -12 < 0 \quad \Delta < 0 \quad \text{نلا } \varepsilon < 0 \end{aligned}$$

$$\frac{-2}{-4+8} \quad \frac{2}{-4+8} \Rightarrow D_f = (-2, 0) \cup (2, +\infty)$$

$$\left. \begin{aligned} f(x) : x=1 &\rightarrow f(1) = 1-1 = 1 \\ f(1.0) : x=1 &\rightarrow f(1.0) = 1-1 = 1 \end{aligned} \right\} f(x) + f(1.0) = 1+1 = 2$$

$$f(x) = x^4 - 4x^2 + 1 \Rightarrow x^4 - 4x^2 + 1 = 1 + 1 \Rightarrow (x-1)^4 + 1$$

$$g(x) = x^4 + 4x^2 + 1 \Rightarrow x^4 + 4x^2 + 1 = 1 + 1 \Rightarrow (x+1)^4 + 1$$

$$\frac{g(\sqrt{x}-1)}{f(\sqrt{x}+1)} = \frac{1-1}{1+1} = \underline{-1}$$

$$f(x) = \sqrt{x+\varepsilon} \sqrt{x-\varepsilon} = \sqrt{(\sqrt{x-\varepsilon} + \varepsilon)^2} = |\sqrt{x-\varepsilon} + \varepsilon| = \sqrt{x-\varepsilon} + \varepsilon$$

$$g(x) = \sqrt{x-\varepsilon} \sqrt{x-\varepsilon} = \sqrt{(\sqrt{x-\varepsilon} - \varepsilon)^2} = |\sqrt{x-\varepsilon} - \varepsilon| = A$$

$$\sqrt{x-\varepsilon} + \varepsilon + |\sqrt{x-\varepsilon} - \varepsilon| = a + b\sqrt{x+c} \quad A > 0 \rightarrow \sqrt{x-\varepsilon} + 0$$

$$\Rightarrow a=0, b=\varepsilon, c=-\varepsilon$$

$$A < 0 \rightarrow a + b\sqrt{x+c} = \varepsilon \begin{cases} a=\varepsilon, b=?; c=-x \\ a=\varepsilon, b=0; c=? \end{cases} \Rightarrow \frac{a+b}{c} = \frac{\varepsilon}{-\varepsilon} = -1$$

$$\frac{g}{f} = \left\{ \left(\frac{1}{\varepsilon}, \frac{\varepsilon}{\varepsilon} \right), (0, 1) \right\}$$

$$a) \left\{ \left(\frac{1}{\varepsilon}, \frac{\varepsilon}{\varepsilon} \right), \left(\frac{1}{\varepsilon}, -1 \right), (1, 1), \left(-\frac{1}{\varepsilon}, 1 \right) \right\}$$

$$b) \{ (1, 1), (\sqrt{2}, -1), (1, 1) \}$$

$$c) \{ (-1, 1), (1, 1), (1, 1), (-1, 1) \}$$

$$d) \{ (1, 1), (1, -1), (-1, \frac{\pi}{2}) \}$$