

$$f(x) = \left(\frac{1}{x}\right)^{x-1} \quad y = \sqrt{x^3 f(x+1)} = \sqrt{x^3 \left(\frac{1}{x}\right)^{x-1}} = 1$$

$$\left(\frac{1}{x}\right)^{x-1} - 1 = 0 \quad x = 1$$

x	0	1		
x^3	-	0	+	+
$\left(\frac{1}{x}\right)^{x-1}$	+	+	0	-
$x^3 \left(\frac{1}{x}\right)^{x-1}$	-	0	+	-

$\text{ج.ر} = [0, 1]$

$$f(x) = \sqrt{x+|x+1|} \rightarrow f(-x) = \sqrt{-x+|1-x|}$$

$$-x+|1-x| \geq 0 \rightarrow |1-x| \geq x \quad \begin{cases} x \leq 1 \\ x > 1 \end{cases}$$

برای $x \leq 1$: $1-x \geq x \rightarrow 1 \geq 2x \rightarrow x \leq \frac{1}{2}$
 برای $x > 1$: $1-x \leq -x \rightarrow 1 \leq 0$ (غیرممکن)

$\text{ج.ر} = (-\infty, \frac{1}{2}]$

$$y = \sqrt{\frac{x-1}{f(x)}}$$

x	-1	0	1	2	3
$x-1$	-	-	0	+	+
$f(x)$	+	0	-	+	0
$\frac{x-1}{f(x)}$	-	+	-	+	+

$\text{ج.ر} = (-1, 0] \cup (2, 3)$

$$9x^4 - 13x^2 - 4x - 3 = (3x^2 + x + 1)(3x^2 - x - 3)$$

$$\Delta = 3^2 - 4 \cdot 3 \cdot (-3) = 3^2 + 36 = 39$$

$$x = \frac{1 \pm \sqrt{39}}{4}$$

$$\frac{1}{3x^2 + ax + b} = \frac{1}{3x^2 - x - 3} \quad a = -1 \quad b = -3$$

$$\sqrt{-(-1-3)} = \sqrt{4} = 2$$

$$f(x) = x^4 + x \quad y = \sqrt{f(x) - f\left(\frac{x}{2}\right)} = \sqrt{x^4 + x - \left(\left(\frac{x}{2}\right)^4 + \frac{x}{2}\right)} = \sqrt{x^4 + x - \frac{x^4}{16} - \frac{x}{2}} = \sqrt{\frac{15x^4}{16} + \frac{x}{2}}$$

$$x^4 + x - \frac{x^4}{16} - \frac{x}{2} = \frac{15x^4}{16} + \frac{x}{2} = \frac{x^4}{16} (15 + 8x) = \frac{x^4}{16} (x+2)(x-2)(x^2 + 8x + 14)$$

x	-2	0	2
$\frac{15x^4}{16} + \frac{x}{2}$	-	0	+

$\text{ج.ر} = [-2, 0) \cup [2, +\infty)$

$$f(x^2+x) = x^2-1$$

$$x=2 \rightarrow f(10)=V$$

$$x=1 \rightarrow f(2)=1$$

$$f(10)+f(2)=V+1=\boxed{A}$$

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$$f(x) = x^3 - 4x^2 + 12x - 1 + 1 = (x-2)^3 + 1 \rightarrow f(\sqrt[3]{2}+2) = (\sqrt[3]{2})^3 + 1 = \boxed{10}$$

$$g(x) = x^3 + 9x^2 + 27x + 27 - 27 = (x+3)^3 - 27 \rightarrow g(\sqrt[3]{27}-3) = (\sqrt[3]{27})^3 - 27 = \boxed{-20}$$

$$\frac{-20}{10} = \boxed{-2}$$

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$$f(x) = \sqrt{x+4}\sqrt{x-4}$$

$$g(x) = \sqrt{x-4}\sqrt{x-4}$$

$$f(x)+g(x) = \sqrt{(x-4)+4}\sqrt{x-4+4} + \sqrt{(x-4)-4}\sqrt{x-4+4} = \sqrt{(x-4+4)^2} + \sqrt{(x-4-4)^2} = \boxed{A}$$

$$|\sqrt{x-4}+2| + |\sqrt{x-4}-2| = \underbrace{2\sqrt{x-4}}_{a+b} \underbrace{1}_{c} \quad \frac{a+b}{c} = \frac{2}{1} = \boxed{2}$$

$$f(x) = \frac{x+2}{x^2-5x+3} \quad g(x) = \{(2,1), (1,0), (0,2), (3,4)\} \quad \frac{g}{f} = ?$$

$$x^2-5x+3=0 \rightarrow x^2=5x-3$$

$$D_{\frac{g}{f}} = \{2, 1, 0, 3\} - \{3\} = \{0, 1, 2\}$$

$$\frac{g}{f} = \left\{ (2, \frac{1}{-1}), (0, \frac{2}{3}) \right\}$$

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$$f(x) = \{(1,2), (3,-1), (4,2), (-1,4)\} \quad g(x) = \{(-2,3), (1,-1), (3,2), (-6,0)\}$$

$$f(2x) = \left\{ \left(\frac{1}{2}, 2\right), \left(\frac{3}{2}, -1\right), (2, 2), \left(-\frac{1}{2}, 4\right) \right\}$$

$$f(x^2) = \{(1,2), (-1,2), (\sqrt{2}, -1), (-\sqrt{2}, -1), (2,2), (-2,2)\}$$

$$g(g(x)) = g(g(1)) = g(0) = 0$$

$$x = -1 \rightarrow g(g(-1)) = g(-1) = 0$$

$$\{(-2,0), (1,1)\}$$

$$x = -2 \rightarrow g(g(-2)) = g(3) = 2$$

$$x = 2 \rightarrow g(g(2)) = g(2) = 2$$

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$$\frac{2f}{g} = \{(1,-4), (3,-1)\}$$

$$D_{\frac{2f}{g}} = \{1, 3\} - \{3\} = \{1\}$$