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$f(x) = \left(\frac{1}{2}\right)^{x-2}$ $\quad y = \sqrt{x^3 f(x+1)}$ (1)

$f(x+1) = \frac{1}{2}^{(x+1)-2} = \left(\frac{1}{2}\right)^{x-1}$

$y = \sqrt{x^3 \times \left(\frac{1}{2}\right)^{x-1}}$

$x^3 \left(\frac{1}{2}\right)^{x-1} \geq 0 \longrightarrow \left(\frac{1}{2}\right)^{x-1} > 0$

$x^3 \geq 0 \longrightarrow x \geq 0 \longrightarrow D_f = [0, +\infty)$

$f(x) = \sqrt{x + |x+2|}$ (2)

$f(-x) = \sqrt{-x + |-x+2|}$

$-x+2 \geq 0 \longrightarrow x \leq 2$

$f(-x) = \sqrt{-x + (-x+2)} = \sqrt{-2x+2}$

$-2x+2 \geq 0 \longrightarrow x \leq 1$

$-x+2 < 0 \longrightarrow x > 2 \Rightarrow f(x) = \sqrt{-x+(x-2)} = \sqrt{-2}$

$D_f = (-\infty, 1]$ $\quad \Downarrow \; x \leq 1$

$y = \sqrt{\frac{x-1}{f(x)}} \longrightarrow f(x) \neq 0 \longrightarrow \frac{x-1}{f(x)} \geq 0$ (3)

$x = 3, 2, -4, -5$

$(-5, -4) \rightarrow f(x) > 0$ ✗

$(-4, 1) \longrightarrow f(x) < 0$ ✓

$(2, 3) \longrightarrow f(x) > 0$ ✓

$D_f = (-4, 1] \cup (2, 3)$

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(۴)

$y = \frac{1}{3x^2+ax+b}$

$9x^2 - 7x^2 - 4x - 3 = 2x^2 - 4x - 3$

$\frac{2}{3} = -\frac{4}{a} = -\frac{3}{b}$ (1) $\frac{2}{3} = \frac{-4}{a} \Rightarrow 2a = -12 \Rightarrow a = -6$

(2) $\frac{2}{3} = -\frac{3}{b} \Rightarrow 2b = -9 \Rightarrow b = -\frac{9}{2}$

$a + b = -6 + (-\frac{9}{2}) = -\frac{12}{2} - \frac{9}{2} = -\frac{21}{2}$

$\sqrt{-(a+b)} = \sqrt{-(-\frac{21}{2})} = \sqrt{\frac{21}{2}}$

$\sqrt{\frac{21}{2}} = \frac{\sqrt{21}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{42}}{2}$

(۵)

$f(x) = x^3 + x$ $\quad y = \sqrt{f(x) f(\frac{4}{x})}$

$f(x) - f(\frac{4}{x}) = (x^3 + x) - ((\frac{4}{x})^3 + \frac{4}{x}) = (x^3 - \frac{64}{x^3}) + (x - \frac{4}{x})$

$x^3 - \frac{64}{x^3} = (x - \frac{4}{x})(x^2 + 4 + \frac{16}{x^2})$

$(x - \frac{4}{x})(x^2 + 4 + \frac{16}{x^2}) + (x - \frac{4}{x}) = (x - \frac{4}{x})(x^2 + 4 + \frac{16}{x^2}$

$= (x - \frac{4}{x})(x^2 + 5 + \frac{16}{x^2})$

$\frac{x^2 - 4}{x} \geq 0 \to \frac{(x-2)(x+2)}{x} \geq 0$

x	-2	0	2	
P	$-$	$+$	$-$	$+$

$D_f = [-2, 0) \cup [2, +\infty)$

(۶)

$f(x^۳+x)=۲x-۱$

$x^۳+x=۱۰$

$x=۲ \rightarrow ۲^۳+۲=۸+۲=۱۰$

$f(۱۰)=۲(۲)-۱=۴-۱=۳$

$x^۳+x=۲ \rightarrow x=۱ \rightarrow ۱^۳+۱=۱+۱=۲$

$f(۲)=۲(۱)-۱=۲-۱=۱ \rightarrow f(۱۰)+f(۲)=۳+۱=۴$

(۷)

$g(x)=x^۳+۹x^۲+۲۷x \qquad t=\sqrt[۳]{۷}-۳$

$g(t)=(\sqrt[۳]{۷}-۳)^۳+۹(\sqrt[۳]{۷}-۳)^۲+۲۷(\sqrt[۳]{۷}-۳)$

$=(۷-۲۷)+۲۷\sqrt[۳]{۷}-۹\sqrt[۳]{۴۹}=-۲۰+۲۷\sqrt[۳]{۷}-۹\sqrt[۳]{۴۹}$

$۹(\sqrt[۳]{۷}-۳)^۲=۹((\sqrt[۳]{۷})^۲-۶\sqrt[۳]{۷}+۹)$

$=۹\sqrt[۳]{۴۹}-۵۴\sqrt[۳]{۷}+۸۱$

$۲۷(\sqrt[۳]{۷}-۳)=۲۷\sqrt[۳]{۷}-۸۱$

$g(t)=(-۲۰+۲۷\sqrt[۳]{۷}-۹\sqrt[۳]{۴۹})+(۹\sqrt[۳]{۴۹}-۵۴\sqrt[۳]{۷}+۸۱)$

$+(۲۷\sqrt[۳]{۷}-۸۱)=-۲۰$

$f(x)=(\sqrt[۳]{۲}+۲)^۳-۶(\sqrt[۳]{۲}+۲)^۲+۱۲(\sqrt[۳]{۲}+۲)$

$f(x)=(۱۰+۶\sqrt[۳]{۴}+۱۲\sqrt[۳]{۲})+(-۶\sqrt[۳]{۴}-۲۴\sqrt[۳]{۲}-۲۴)+(۱۲\sqrt[۳]{۲}+۲۴)$

$۱۰-۲۴+۲۴=۱۰$

$\frac{g(x)}{f(x)}=-\frac{۲۰}{۱۰}=-۲$

٨

$f(x) = \sqrt{x + ٤\sqrt{x - ٤}}$

$f(x) + g(x) = \sqrt{x - ٤} + ٢ + \sqrt{x - ١}$

٩

$f(x) = \dfrac{x+٢}{(x-١)(x-٣)}$

$x = ١ \rightarrow$ صفر $= ٠$

$x = ٢ \longrightarrow \dfrac{٢+٢}{(٢-١)(٢-٣)} = \dfrac{٤}{١(-١)} = -٤$

$\dfrac{g(٢)}{f(٢)} = \dfrac{١}{-٤} = -\dfrac{١}{٤}$

$x = ٣ \rightarrow f(x) = ٠$

$x = ٤ \Rightarrow f(٤) = \dfrac{٤+٢}{(٤-١)(٤-٣)} = \dfrac{٦}{٣(١)} = ٢$ $\qquad \dfrac{g(٤)}{f(٤)} = \dfrac{٠}{٢} = ٠$

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$f(x) = \{(-1,6),(1,2),(3,-1),(2,4)\}$ ۱۰

$g(x) = \{(-2,3),(1,-1),(3,2),(-1,0)\}$

الف) $f = -1, 1, 3, 4$

$2x = -1 \Rightarrow x = -\frac{1}{2}$

$2x = 1 \Rightarrow x = \frac{1}{2}$

$2x = 3 \Rightarrow x = \frac{3}{2}$

$2x = 4 \Rightarrow x = 2$

$f(2x) = \{(-\frac{1}{2}, 6),(\frac{1}{2}, 2),(\frac{3}{2}, -1),(2,2)\}$

ب) $f(x^2)$

$x^2 = 1 \rightarrow x = \pm 1 \rightarrow f(1) = 2$

$x^2 = 3 \Rightarrow x = \pm\sqrt{3} \rightarrow f(3) = -1$

$x^2 = 4 \Rightarrow x = \pm 2 \Rightarrow f(4) = 2$

$f(x^2) = \{(-2,2),(-\sqrt{3},-1),(-1,2),(1,2),(\sqrt{3},-1),(2,2)\}$

ج) $2g^2(x)+1$

$g^2(-2) = 2\times 9+1 = 19$

$g^2(1) = (-1)^2 = 1 \Rightarrow 2\times 1+1 = 3$

$g^2(3) = 2^2 = 4 \rightarrow 2\times 4+1 = 9$

$g^2(-1) = 0^2 = 0 \rightarrow 2\times 0+1 = 1$

$2g^2(x)+1 = \{(-2,19),(1,3),(3,9),(-1,1)\}$

د) $\frac{f}{g} \Rightarrow -1, 1, 3 \rightarrow \frac{f}{g} = \{(1,-2),(3,-\frac{1}{2})\}$

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$$f(x) = \sqrt{x-4\sqrt{x-4}} \qquad (8)$$

$$g(x) = \sqrt{x-4\sqrt{x-4}}$$

$$(f(x) + g(x)) = a + b\sqrt{x+c}$$

$$(x+4\sqrt{x-4}) + (x-4\sqrt{x-4}) + 2\sqrt{(x+4\sqrt{x-4})(x-4\sqrt{x-4})}$$

$$2x + 2(x+8) = 4x+16$$

$$f(x) + g(x) = \sqrt{4x+16} = 2\sqrt{x+4}$$

$$a=0 \quad b=2 \quad c=4 \longrightarrow \frac{x+b}{c} = \frac{2}{4} = \frac{1}{2}$$