۱

الف) $y = x + |2x - 4|$

$\Rightarrow y = \begin{cases} x + 2x - 4 & x \geq 2 \\ 2 & x = 2 \\ x - 2x + 4 & x < 2 \end{cases}$

$\Rightarrow \begin{cases} 3x - 4 & x \geq 2 \Rightarrow R_f = [2, +\infty) \\ 2 & x = 2 \Rightarrow 2 \\ -x + 4 & x < 2 \Rightarrow R_f = [2, +\infty) \end{cases} \Rightarrow R_f = [2, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$

$\begin{cases} x - 2 + x - 1 - x & x \geq 2 \\ -x + 2 + x - 1 - x & x \geq 1 \\ -x + 2 - x + 1 - x & x \geq 0 \end{cases}$

$\Rightarrow \begin{cases} x - 3 & x \geq 2 \\ -x + 1 & x \geq 1 \\ -3x + 3 & x \geq 0 \end{cases} \Rightarrow R_f = [-1, +\infty)$

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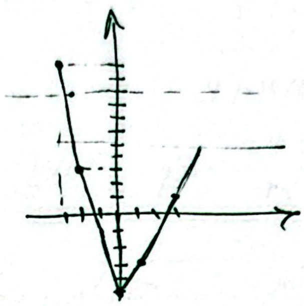
$|3x - 3| - |x + 2| = K \quad K = ?$

$x = 1 \quad x = -2$

$\begin{cases} -3x + 3 + x + 2 & x < -2 \\ -3x - 3 - x - 2 & -2 \leq x < 1 \\ 3x - 3 - x - 2 & x \geq 1 \end{cases}$

$\Rightarrow \begin{cases} -2x + 5 & x < -2 \Rightarrow (9, +\infty) \\ -4x - 5 & -2 \leq x < 1 \Rightarrow [-3, 9] \\ 2x - 5 & x \geq 1 \Rightarrow [-3, +\infty) \end{cases} \Rightarrow R_f = [-3, 9]$



$K = 9$

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الف) $x + \frac{x}{|x|} = y \quad x \neq 0$

$|x| = x$: if $x > 0 \Rightarrow x + \frac{x}{x} = x + 1 \Rightarrow R_f = [1, +\infty)$

$|x| = -x$: if $x < 0 \Rightarrow x + \frac{x}{-x} = x - 1 \Rightarrow R_f = (-\infty, -1]$

$R_f = (-\infty, -1] \cup [1, +\infty)$

ب) $y = x|x| - \frac{x}{|x|} \quad x \neq 0$

$x > 0,\ |x| = x$: $= x^2 - 1 \quad R_f = [0, +\infty)$

$x < 0,\ |x| = -x$: $= -x^2 + 1 \quad R_f = (-\infty, 0]$

$R_f = \mathbb{R}$

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الف) $y = [x] - 2x$

$D_f = \mathbb{R}$

$[-2, 2]$ در بازه

⇓

$D_f = [-2, 2]$

ب) $[x]|x|$

$R_f = \mathbb{R}$

در بازه $[-2, 2]$

⇓

$D_f = [-4, 4]$

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الف) $y = 3x - [3x]$

$R_f = [0, 1)$

ب) $y = 4x - 4[x] = 4(x - [x]) \Rightarrow R_f = [0, 4)$

ج) $x - 5\left[\frac{x}{5}\right] = y$

$R_f = [0, 5)$

د) $y = [2x] - 2x \Rightarrow R_f = (-1, 0]$

$\sin^2 x + \cos^2 x = 1 \Big/ -\sqrt{a^2+b^2} \leq R_f \leq \sqrt{a^2+b^2}$

-۱

الف) $y = 3\sin x + 4\cos x$

$-1 < \sin x, \cos x < 1 \Rightarrow R_f = [-\sqrt{25}, \sqrt{25}]$

ب) $y = 5\sin x - 3\cos x$

$-1 < \sin x, \cos x < 1 \Rightarrow R_f = [-\sqrt{34}, \sqrt{34}]$

ج) $y = [2\sin x + 5\cos x]$

$R_f = [-7, 7]$

د) $\sqrt{\cos x - 2\sin x}$

$\cos x - 2\sin x \geq 0 \Leftarrow \frac{\cos x}{\sin x} \geq 2 \Leftarrow \sin x \neq 0$

$R_f = \sqrt{[-\sqrt{5}, \sqrt{5}]} = [0, \sqrt{\sqrt{5}}]$

۶

الف) $y = \sin^2 x + 3\sin x + 2$

$\max \Rightarrow \sin x = 1 \Rightarrow 6$

$\min \Rightarrow \sin x = -1 \Rightarrow 0$ $\Rightarrow R_f = [0, 6]$

ب) $2\sin^2 x + 3\sin x + 2$

$\max \Rightarrow \sin x = 1 \Rightarrow 7$

$\min \Rightarrow \sin x = -1 \Rightarrow 1$ $\Rightarrow R_f = [1, 7]$

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الف) $\sin^2 x + 4\cos^2 x$ $\qquad \sin^2 x + \cos^2 x = 1$

$y = 1 + \underbrace{3\cos^2 x}_{[0,3]} \Rightarrow R_f = [1, 4]$

ب) $\frac{2\cot x}{1+\cot^2 x} = \frac{2\frac{\cos}{\sin}}{\frac{1}{\sin^2 x}} = 2\cos x \sin x = \sin 2x$ $\qquad R_f = [-1, 1]$

۸

الف) $y = \sin^4 x + \cos^4 x$

$\underbrace{2^{1-2}}_{2^{-1}} \leq \sin^4 x + \cos^4 x \leq 1 \Rightarrow \frac{1}{2} \leq \sin^4 x + \cos^4 x \leq 1 \qquad R_f = [\frac{1}{2}, 1]$

ب) $[\sin^8 x + \cos^8 x]$

$2^{1-4} \leq \sin^8 x + \cos^8 x \leq 1 \Rightarrow \frac{1}{8} \leq \sin^8 x + \cos^8 x \leq 1 \qquad R_f = \{0, 1\}$

۹

الف) $y = \frac{2x^2 + 7x - 4}{x+4} \Rightarrow y = 2x - 1 \qquad , x \neq -4$

$R_f = \mathbb{R}$

ب) $y = \frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2+x+1)}{x-1} \Rightarrow (x+\frac{1}{2})^2 - \frac{1}{4} + 1 = \frac{3}{4} = 0/75$

$-\frac{b}{2a} = -\frac{1}{2}$ $\qquad R_f = [0/75, +\infty)$

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