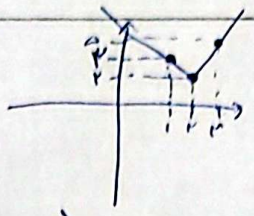
نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۷ ..... کلاس .....

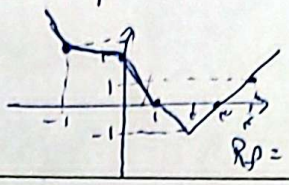
۱

الف) $y = x + |2x-4| \Rightarrow y = \begin{cases} 3x-4 & x \geq 2 \\ -x+4 & x < 2 \end{cases}$



$R_f = [3, +\infty)$

ب) $y = |x-1| + |x-2| - |x| \Rightarrow \begin{cases} -x+3 & x < 0 \\ -3x+3 & 0 \leq x < 1 \\ -x+1 & 1 \leq x < 2 \\ x-3 & 2 \leq x \end{cases}$



$R_f = [-1, +\infty)$

۲

$K = |2x-3| - |x+1|$

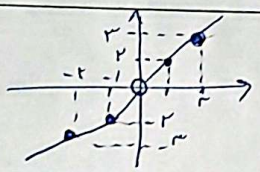
$3x - 3 = -x - 2 \Leftrightarrow -2x = 1 \Rightarrow x = -\frac{1}{2} \Rightarrow K = |2 \times (-\frac{1}{2}) - 3| - |-\frac{1}{2} + 1| = 0$

$K \geq 0$

۳

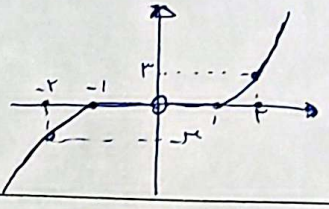
الف) $y = x + \frac{x}{|x|}$

$\begin{cases} x < 0 & y = x - 1 \\ x > 0 & y = x + 1 \end{cases}$



ب) $y = x|x| - \frac{x}{|x|}$

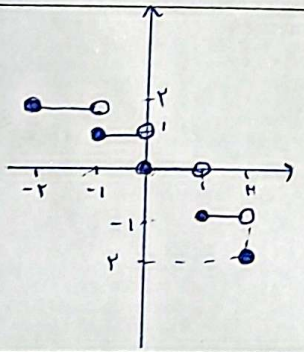
$\begin{cases} x < 0 & y = -x^2 + 1 \\ x > 0 & y = x^2 - 1 \end{cases}$



۴

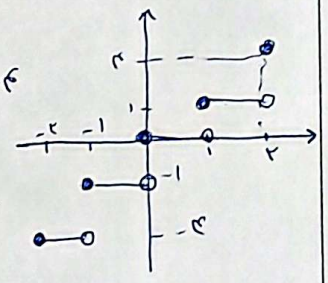
الف) $y = [x] - 2x$

$\begin{cases} -2 \leq x < -1 \Rightarrow y = 2 \\ -1 \leq x < 0 \Rightarrow y = 0 \\ 0 \leq x < 1 \quad y = x \\ 1 \leq x < 2 \quad y = -1 \\ x = 2 \quad y = 2 \end{cases}$



ب) $y = [x]|x|$

$\begin{cases} -4 \leq x < -3 \Rightarrow y = -4 \\ -3 \leq x < -2 \quad y = -3 \\ -2 \leq x < -1 \quad y = -2 \\ -1 \leq x < 0 \quad y = 0 \\ 0 \leq x < 1 \quad y = 0 \\ 1 \leq x < 2 \quad y = 1 \\ x = 2 \quad y = 4 \end{cases}$



۵

الف) $y = 2x - [2x] \quad R_f = [0, 1)$

ب) $y = 4x - 4[x] \Rightarrow y = 4(x - [x]) \Rightarrow R_f = [0, 4)$

ج) $y = x - 5\left[\frac{x}{5}\right] \Rightarrow y = 5\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right) \Rightarrow R_f = [0, 5)$

د) $y = [2x] - 2x \Rightarrow 2x \in [2x] \Rightarrow [2x] - 2x \geq 0 \Rightarrow R_f = (-1, 0]$

٦ أ) $y = 3\sin x + 4\cos x \Rightarrow -\sqrt{3^2+4^2} \le 3\sin x + 4\cos x \le \sqrt{3^2+4^2}$ $\boxed{R_f = [-\sqrt{25}, \sqrt{25}]}$

ب) $y = 5\sin x - 12\cos x \Rightarrow -\sqrt{169} \le 3\sin x - 12\cos x \le \sqrt{169}$ $\boxed{R_f = [-13, 13]}$

ج) $y = [3\sin x + 5\cos x] \Rightarrow -\sqrt{34} \le 3\sin x + 5\cos x \le \sqrt{34} \Rightarrow [3\sin x + 5\cos x] = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

$\boxed{R_f = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}}$

د) $y = [\cos x - 2\sin x]$, $0 \le x \le \frac{\pi}{2}$ ... $\boxed{R_f = [0, \sqrt{2}]}$

٧ أ) $y = 2\sin^2 x + 2\sin x + 2$

$\sin x = 1 \rightarrow f(1) = 6$ max

$\sin x = -1 \rightarrow f(-1) = 2$ min

$\sin x = \frac{-b}{2a} = -\frac{2}{4} = -\frac{1}{2} \Rightarrow f(-\frac{1}{2}) = \frac{3}{2}$ min

$-1 \le \sin x \le 1$

$\boxed{R_f = [\frac{3}{2}, 6]}$

ب) $y = 2\sin^2 x + 2\sin x + 2$

$\sin x = 1 \rightarrow f(1) = 6$ max

$\sin x = -1 \rightarrow f(-1) = 2$

$\sin x = \frac{-b}{2a} = -\frac{2}{4} = -\frac{1}{2} \rightarrow f(-\frac{1}{2}) = \frac{3}{2}$ min

$\boxed{R_f = [\frac{3}{2}, 6]}$

٨ أ) $y = \sin^2 x + 4\cos^2 x \Rightarrow y = \sin^2 x + 4(1 - \sin^2 x) \Rightarrow y = -3\sin^2 x + 4$

$-1 \le \sin x \le 1 \Rightarrow \boxed{R_f = [1, 4]}$

ب) $y = \frac{2\cot x}{1 + \cot^2 x} \Rightarrow y = \frac{2\cot x}{\frac{1}{\sin^2 x}}$ (cos/sin) $\Rightarrow 2\cos x \sin x = 2 \times (\frac{1}{2}\sin 2x) = \sin 2x$

$\boxed{R_f = [-1, 1]}$

٩ أ) $y = \sin^4 x + \cos^4 x \Rightarrow \frac{1}{2} \le \sin^4 x + \cos^4 x \le 1 \Rightarrow \boxed{R_f = [\frac{1}{2}, 1]}$

ب) $y = [\sin^8 x + \cos^8 x] \Rightarrow \frac{1}{8} \le \sin^8 x + \cos^8 x \le 1 \Rightarrow [\sin^8 x + \cos^8 x] = \{0, 1\}$

$\boxed{R_f = [0, 1]}$

١٠ أ) $y = \frac{2x^2 + 7x - 4}{x + 4} = \frac{x^2 + 7x - 8}{x + 4} = \frac{(x+8)(x-1)}{x+4} = 2x - 1$ if $x \ne -4$ $y = -9$

$\boxed{R_f = \mathbb{R} - \{-9\}}$

ب) $y = \frac{x^3 - 1}{x - 1} = \frac{(x-1)(x^2 + 1 + x)}{x-1} = x^2 + x + 1$

$x = \frac{-b}{2a} = \frac{-1}{2}$ $y = \frac{3}{4}$

if $x = 1$ $y = 3$

$\boxed{R_f = [\frac{3}{4}, +\infty) - \{3\}}$