Subject ۱۴۰۴/۶/۳۰
Date

« به نام خدا »

۱۸,۵

فاطمه متقی - ۷

۱) الف) $y = x + |2x - 4|$

$x - 2x + 4$	$x + 2x - 4$
$-x + 4$	$3x - 4$

$R_f = [2, +\infty)$

ب) $y = |x - 2| + |x - 1| - |x|$

$-x + 2 - x + 1 + x$	$-x + 2 - x$	$-x + 2 + x + 1$	$x - 2 + x - 1 - x$
$-x + 3$	$-x + 1$	$-x$	$x - 3$
	$-2x + 2$	$-x + 1$	

$R_f = [-1, +\infty)$

۲) $y = |3x - 3| - |x + 2|$

$-3x + 3 + x + 2$	$-3x + 3 - x$	$3x - 3 - x - 2$
$-2x + 5$	$-4x + 1$	$2x - 5$

برای اینکه فقط یک ریشه ← فقط در یک نقطه برابر k

$k = 3$

۳) الف) $y = x + \frac{x}{|x|}$

$x - 1$	$x + 1$

ب) $y = x|x| - \frac{x}{|x|}$

$-x^2 + 1$	$x^2 - 1$

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۴) الف) $y = [x] - 2x$

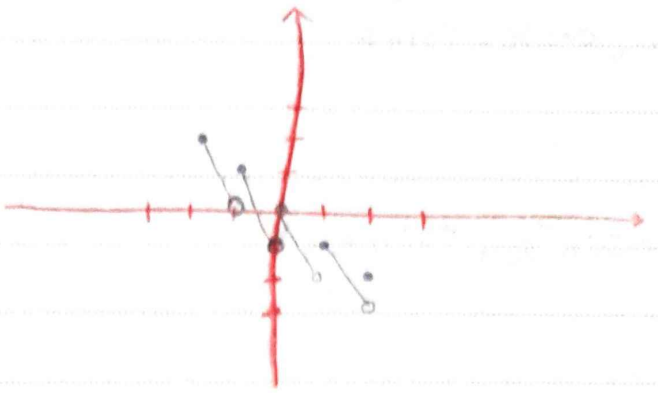
$-2 \le x < -1 \rightsquigarrow -2x - 2$

$-1 \le x < 0 \rightsquigarrow -2x - 1$

$0 \le x < 1 \rightsquigarrow -2x$

$1 \le x < 2 \rightsquigarrow 1 - 2x$

$x = 2 \rightsquigarrow -4$



ب) $y = [x]\,|x|$

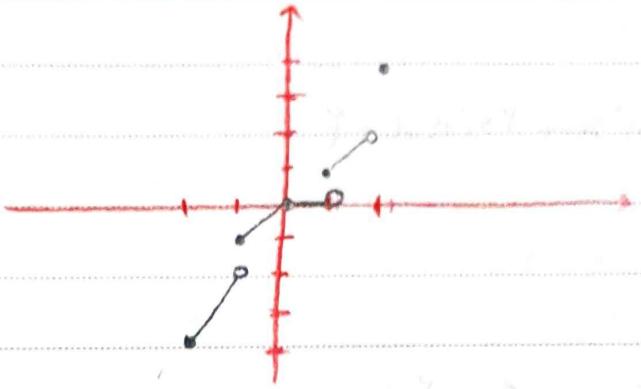
$-2 \le x < -1 \rightsquigarrow 2x$

$-1 \le x < 0 \rightsquigarrow x$

$0 \le x < 1 \rightsquigarrow 0$

$1 \le x < 2 \rightsquigarrow x$

$x = 2 \rightsquigarrow 4$



۵) $0 \le x - [x] < 1$

الف) $y = 3x - [3x] \rightsquigarrow R_f = [0, 1)$

ب) $y = 4x - 4[x] \rightsquigarrow 4(x - [x]) \Rightarrow R_f = [0, 4]$

ج) $y = x - 5\left[\frac{x}{5}\right] \rightsquigarrow 5\left(\frac{x}{5} - \left[\frac{x}{5}\right]\right) \Rightarrow R_f = [0, 5]$

د) $y = [2x] - 2x \rightsquigarrow -(2x - [2x]) \Rightarrow R_f = (-1, 0]$

۶) الف) $y = 3\sin x + 2\cos x \rightarrow -\sqrt{3^2 + 2^2} \le 3\sin x + 2\cos x \le \sqrt{3^2 + 2^2}$

$\hookrightarrow R_f = [-\sqrt{13}, \sqrt{13}]$ (1/75)

ب) $y = 4\sin x - 3\cos x \rightarrow -\sqrt{4^2 + (-3)^2} \le 4\sin x - 3\cos x \le \sqrt{4^2 + (-3)^2}$

$\hookrightarrow -\sqrt{25} \le 4\sin x - 3\cos x \le \sqrt{25} \Rightarrow R_f = [-5, 5]$

ج) $y = [2\sin x + 5\cos x] \rightarrow -\sqrt{2^2 + 5^2} \le 2\sin x + 5\cos x \le \sqrt{2^2 + 5^2}$

$-6 \le [2\sin x + 5\cos x] \le 5 \qquad R_f = [-6, 5]$ ← فقط اعداد صحیح

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ج) $y=\sqrt{\cos x-2\sin x} \rightarrow -\sqrt{1^2+(-2)^2} \le \cos x-2\sin x \le \sqrt{1^2+(-2)^2}$

$0 \le \sqrt{\cos x-2\sin x} \le \sqrt[4]{5} \rightarrow R_f=[0,\sqrt[4]{5}]$

۷) الف) $y=\sin^2 x+3\sin x+2 \quad R_f=[0,6]$

$\hookrightarrow \sin x=-1 \rightsquigarrow 1-3+2=0$

$\hookrightarrow \sin x=1 \rightsquigarrow 1+3+2=6$

$\hookrightarrow \sin x=\frac{-b}{a}=-\frac{3}{2} \rightarrow \frac{9}{4}-\frac{9}{2}+2=\frac{1}{4}$

1,0

ب) $y=2\sin^2 x+3\sin x+2 \quad R_f=[\frac{1}{8},7]$

$\hookrightarrow \sin x=-1 \rightarrow 2-3+2=1$

$\hookrightarrow \sin x=1 \rightarrow 2+3+2=7$

$\hookrightarrow \sin x=\frac{-b}{a}=\frac{-3}{4} \rightarrow 2\times\frac{9}{16}-\frac{9}{4}+2=\frac{1}{8}$

۸) الف) $y=\sin^2 x+4\cos^2 x \rightarrow \sin^2 x+4(1-\sin^2 x) \quad R_f=[1,4]$

$\hookrightarrow \sin^2 x-4\sin^2 x+4=-3\sin^2 x+4 \Rightarrow 0\le \sin^2 x\le 1$

$\xrightarrow{\times(-3)} -3\le -3\sin^2 x\le 0 \xrightarrow{+4} 1\le -3\sin^2 x+4\le 4$

ب) $y=\frac{2\cot x}{1+\cos^2 x} \rightarrow \frac{2\frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}}=2\sin x\cos x=\sin 2x$

$-1\le \sin 2x\le 1 \rightarrow R_f=[-1,1]$

۹) $2^{1-k} \le \sin^{2k} x+\cos^{2k} x \le 1$

الف) $y=\sin^4 x+\cos^4 x$

$2^{1-3}\le \sin^2 x+\cos^4 x\le 1 \Rightarrow R_f=[\frac{1}{2},1]$

ب) $[\sin^8 x+\cos^8 x] \Rightarrow \underbrace{2^{1-4}}_{\frac{1}{8}} \le \sin^8 x+\cos^8 x\le 1$

داخل براکت برود

$R_f=\{0,1\}$

الف) $y = \frac{2x^2 + 7x - 4}{x + 4} = 2x - 1$

$2x^2 + 7x - 4$	$x + 4$
$-2x^2 - 8x$	$2x - 1$
$-x - 4$	
$+x + 4$	
0	

(۱۰

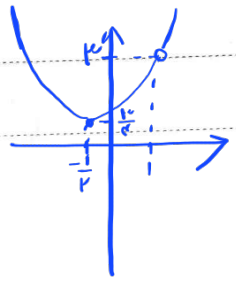
$x \neq -4 \rightarrow y \neq 2(-4) - 1 = -9$

1,50

$\hookrightarrow R_f = \mathbb{R} - \{-9\}$

ب) $y = \frac{x^3 - 1}{x - 1} \rightsquigarrow \frac{(x-1)(x^2 + x + 1)}{x - 1} \Rightarrow x^2 + x + 1$

$x \neq 1 \rightarrow y \neq 1 + 1 + 1 = 3 \longrightarrow R_f = \mathbb{R} - \{3\}$



$R = [\frac{3}{4}, +\infty)$