

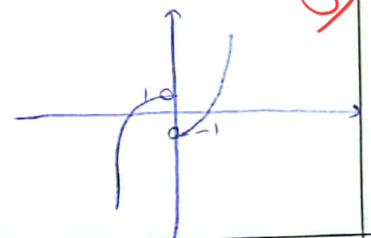
الف) $y = x + |2x - 4|$ $x=2$
 $\begin{cases} x \geq 2 & 3x - 4 \rightarrow R = [2, +\infty) \\ x < 2 & -x + 4 \rightarrow R = (4, +\infty) \end{cases} \cup \rightarrow [2, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$
 $\begin{array}{l} x-2+1-x = -1 \rightarrow R = (-1, +\infty) \\ x-2+x = x-2 \rightarrow R = [2, +\infty) \\ -x+1-x = -2x+1 \rightarrow R = (-\infty, 1] \end{array}$
 $\bullet \cup (-1, +\infty) \cup [2, +\infty) \cup (-\infty, 1] = (-1, +\infty)$

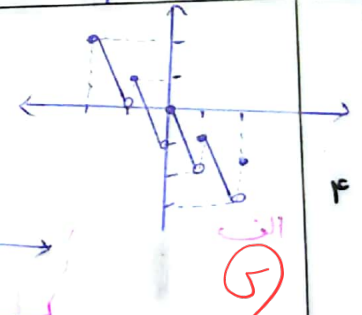
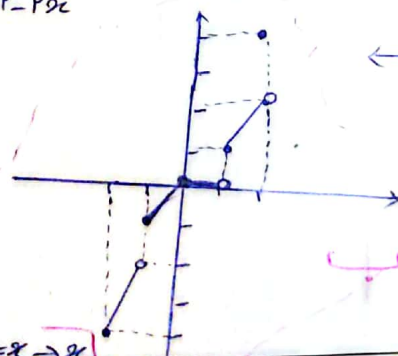
$|3x-3| - |x+2| = k$
 $\begin{cases} 3x-3-x-2 = k \rightarrow 2x-5 = k \rightarrow R = [-5, +\infty) \\ 3x-3+x+2 = k \rightarrow 4x-1 = k \rightarrow R = [1, +\infty) \end{cases}$

ریشه داریم چون $x = -2 \rightarrow k = 9$
 در این نقاطی از نمودارها منفرد است. $x = 1 \rightarrow k = -3$

الف) $y = x + \frac{x}{|x|}$ $\begin{cases} x > 0 \rightarrow x+1 \\ x < 0 \rightarrow x-1 \end{cases}$
 ب) $y = x|x| - \frac{x}{|x|}$ $\begin{cases} x > 0 \rightarrow x^2-1 \\ x < 0 \rightarrow -x^2+1 \end{cases}$



الف) $y = [x] - 2x$ $[-2, -1) \rightarrow [x] = -2 \rightarrow -2-2x$
 $[-1, 0) \rightarrow [x] = -1 \rightarrow -1-2x$
 $[0, 1) \rightarrow [x] = 0 \rightarrow -2x$
 $[1, 2) \rightarrow [x] = 1 \rightarrow 1-2x$
 ب) $[x] |x|$
 $[-2, -1) \rightarrow [x] = -2, |x| = -x \rightarrow 2x$
 $[-1, 0) \rightarrow [x] = -1, |x| = -x \rightarrow x$
 $[0, 1) \rightarrow [x] = 0 \rightarrow 0$
 $[1, 2) \rightarrow [x] = 1, |x| = x \rightarrow x$



الف) $y = 3x - [3x] \rightarrow 0 \leq 3x - [3x] < 1 \rightarrow R = [0, 1)$

$0 \leq \heartsuit - [\heartsuit] < 1$
 $-1 < [\heartsuit - \heartsuit] \leq 0$

ب) $y = 4x - 4[x] \rightarrow 4(x - [x]) \rightarrow R = [0, 4)$

ج) $y = x - 5[\frac{x}{5}] = 5(\frac{x}{5} - [\frac{x}{5}]) \rightarrow R = [0, 5)$

د) $y = [2x] - 2x \rightarrow R = [-1, 0]$

الف) $y = 3\sin x + 4\cos x \rightarrow -\sqrt{4+9} \leq 3\sin x + 4\cos x \leq \sqrt{4+9}$ $-\sqrt{a^2+b^2} \leq a\sin x + b\cos x \leq \sqrt{a^2+b^2}$

$R = [-\sqrt{17}, \sqrt{17}]$

ب) $y = 4\sin x - 3\cos x \rightarrow -5 \leq 4\sin x - 3\cos x \leq 5$ $R = [-5, 5]$

ج) $y = [r\sin x + 5\cos x] \rightarrow -\sqrt{r^2+25} \leq r\sin x + 5\cos x \leq \sqrt{r^2+25}$

$R = [-\sqrt{r^2+25}, \sqrt{r^2+25}]$ $\rightarrow -\sqrt{a} \leq \cos x - 3\sin x \leq \sqrt{a}$
 $\sqrt{a} \leq \cos x - 3\sin x \leq \sqrt{a}$

الف) $y = \sin^2 x + 3\sin x + 4$ $R = [0, 4]$

$\min \sin^2 = -1 \rightarrow 1 - 3 + 4 = 2 \rightarrow \min$
 $\max \sin^2 = 1 \rightarrow 1 + 3 + 4 = 8 \rightarrow \max$
 $-\frac{b}{2a} = -\frac{3}{2} \rightarrow y = \frac{9}{4} - \frac{9}{4} + 4 = 4$

ب) $y = 3\sin^2 x + 3\sin x + 4$ $R = [\frac{1}{4}, 4]$

$\min \sin^2 = -1 \rightarrow 3 - 3 + 4 = 4 \rightarrow \min$
 $\max \sin^2 = 1 \rightarrow 3 + 3 + 4 = 10 \rightarrow \max$
 $-\frac{b}{2a} = -\frac{3}{6} = -\frac{1}{2} \rightarrow y = \frac{9}{4} \times \frac{1}{4} - \frac{9}{4} + 4 = \frac{1}{4}$

الف) $\sin^2 x + 4\cos^2 x = 1 - \cos^2 x + 4\cos^2 x = 3\cos^2 x + 1 \rightarrow 0 \leq \cos^2 x \leq 1$
 $0 \leq 3\cos^2 x \leq 3$
 $1 \leq 1 + 3\cos^2 x \leq 4$ $R = [1, 4]$

ب) $y = \frac{4\cos^2 x}{1 + \cos^2 x} = \frac{4 \frac{\cos^2 x}{\sin^2 x}}{1 + \frac{\cos^2 x}{\sin^2 x}} = 4 \cos^2 x \sin^2 x = \sin^2 x \rightarrow -1 \leq \sin^2 x \leq 1$
 $R = [-1, 1]$

الف) $y = \sin^4 x + \cos^4 x \xrightarrow{k=3} \frac{1}{4} \leq \sin^4 x + \cos^4 x \leq 1$ $R = [\frac{1}{4}, 1]$

ب) $y = [\sin^4 x + \cos^4 x] \xrightarrow{k=4} \frac{1}{16} \leq \sin^4 x + \cos^4 x \leq 1 \rightarrow \frac{1}{16} \leq \sin^4 x + \cos^4 x \leq 1$
 $R = [\frac{1}{16}, 1]$

الف) $y = \frac{x^2 + 4x - 4}{x + 4} \rightarrow x^2 + 4x - 4 = \frac{(x+4)^2 + V(x) - 1}{x} = \frac{(x+4)(x-1)}{x}$ $R = [0, 1]$

$(x+4)(x-1) \rightarrow y = \frac{(x+4)(x-1)}{x+4} = x-1 \xrightarrow{x=-4} y = -5$ $R = [0, 1]$

ب) $y = \frac{x^2 - 1}{x - 1} = \frac{(x-1)(x+1)}{x-1} = x+1 \rightarrow -\frac{b}{2a} = -\frac{1}{2} \rightarrow y = \frac{1}{4} - \frac{1}{4} + 1 = \frac{3}{4}$
 $x=1 \rightarrow y=2$ $R = [\frac{3}{4}, +\infty)$