نام و نام خانوادگی ..... سبا فیضی ..... پاسخنامه تشریحی تکلیف شماره ..... ۷ ..... کلاس گروه یازدهم دختر B

۱

الف) $y = x + |2x-4|$

۱) $2x-4 \geq 0 \Rightarrow 2x \geq 4 \Rightarrow x \geq 2$

$x + 2x - 4 \Rightarrow 3x - 4$

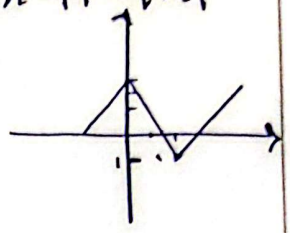
۲) $2x-4 < 0 \Rightarrow 2x < 4 \Rightarrow x < 2$

$x - 2x + 4 \Rightarrow 4 - x$

$R_y = [2, +\infty)$

ب) $y = |x-2| + |x-1| - |x|$

$$\begin{cases} x-3 & 2 \leq x \\ 3x-3 & 1 \leq x < 2 \\ x-1 & 0 \leq x < 1 \\ 3-x & x < 0 \end{cases}$$



$R_y = [-1, +\infty)$

۲

$|3x-3| - |x+2| = k$

۱) $3x-3=0 \Rightarrow x=1$

۲) $x+2=0 \Rightarrow x=-2$

$(1, +\infty), [-2, 1], (-\infty, -2)$

$k = ?$ معادله یک ریشه دارد!

$$\begin{cases} 5-2x & x > -2 \\ 1-4x & -2 \leq x \leq 1 \\ 5-2x & x > 1 \end{cases}$$

$x = -2 \rightarrow -4-3 = -9 = k$

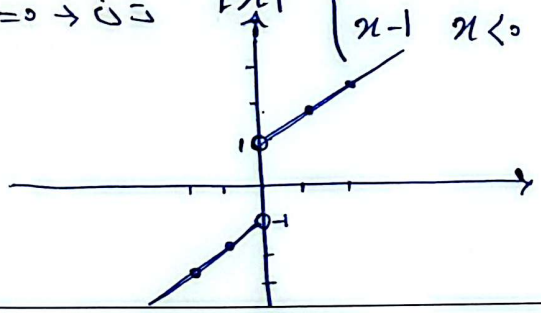
$x = 1 \rightarrow -(1+2) = -3 = k$

۳

الف) $y = x + \frac{x}{|x|}$

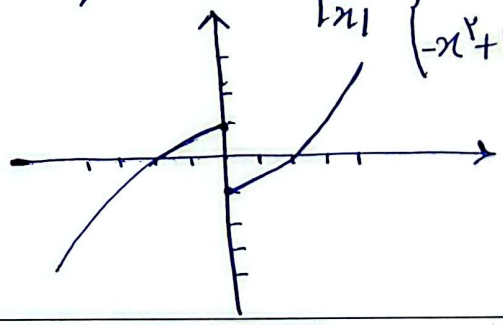
$$\begin{cases} x+1 & x > 0 \\ x-1 & x < 0 \end{cases}$$

$x = 0 \rightarrow$ ت ن



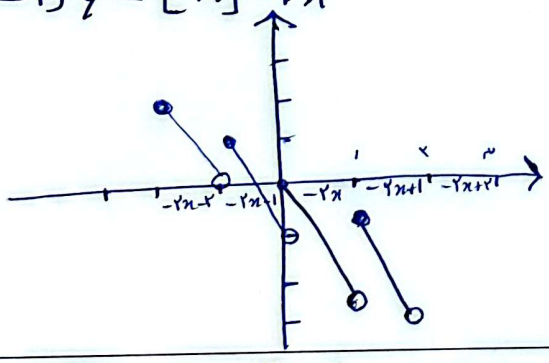
ب) $y = x|x| - \frac{x}{|x|}$

$$\begin{cases} x^2-1 & x > 0 \\ -x^2+1 & x < 0 \end{cases}$$



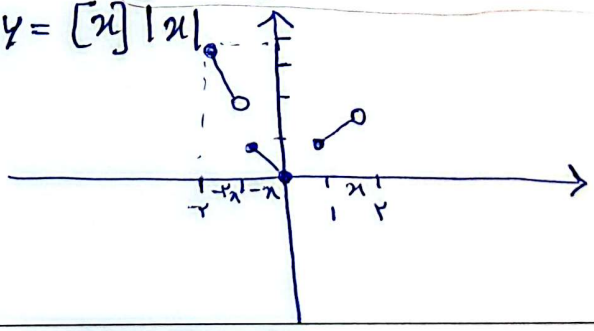
۴

الف) $y = [x] - 2x$



نمودار تابع در بازه $[-2, 2]$

ب) $y = [x]|x|$



۵

الف) $y = 3x - [3x] \Rightarrow 3(x - [x]) \rightarrow R_y = [0, 3)$

ب) $y = 4x - 4[x] \Rightarrow [0, 4)$

ج) $y = x - 5\left[\frac{x}{5}\right] \Longrightarrow R_y = [0, 5)$

د) $y = [2x] - 2x \Rightarrow 2([x] - x) \xrightarrow{\times 2} -2 < [x] - x \leq 0 \Rightarrow R_y = \{-2, 0\}$

۶

الف) $y = 3\sin x + 2\cos x \Rightarrow -\sqrt{9+4} < \quad < \sqrt{9+4} \Rightarrow R_y = [-\sqrt{13}, \sqrt{13}]$

ب) $y = 4\sin x - 3\cos x \Rightarrow -\sqrt{16+9} < \quad < \sqrt{16+3} \Rightarrow R_y = [-5, 5]$

ج) $y = [2\sin x + 5\cos x]$

$-\sqrt{29} < \quad < \sqrt{29} \quad [-\sqrt{29}, \sqrt{29}]$

$R_y = \{5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, -6\}$

د) $y = \sqrt{\cos x - 2\sin x}$

$0 \leq \cos x - 2\sin x \Rightarrow \cancel{-}\sqrt{5} < \quad < \sqrt{5}$

$R_y = [0, \sqrt{5}]$

۷

الف) $y = \sin^2 x + 3\sin^2 x + 2$

$y = 3\sin^2 x + \sin^2 x + 2$

$\sin^2 x$: $0 \to 2$; $1 \to 3 + 1 + 2 = 6$; $\frac{-b}{2a} \to \frac{-1}{6} \to y = \frac{5}{3}$

$R_y = [\frac{5}{3}, 6]$

ب) $y = 2\sin^2 x + 3\sin x + 2$

$\sin x$: $1 \to 2 + 3 + 2 = 7$; $-1 \to 2 - 3 + 2 = 1$; $\frac{-b}{2a} \to \frac{-3}{4} \to y = \frac{7}{8}$

$R_y = [\frac{7}{8}, 7]$

۸

الف) $y = \sin^2 x + 4\cos^2 x$

$1 - \sin^2 x = \cos^2 x$

$\sin^2 x - 4\sin^2 x + 4 \Rightarrow 4 - 3\sin^2 x$

$4\sin^2 x - \sin^2 x + 4 = y$

$0 \leq \sin^2 x \leq 1 \Rightarrow$ 1) $\sin^2 x = 0 \quad y = 4$; 2) $\sin^2 x = 1 \quad y = 0$

$R_y = [1, 4]$

ب) $y = \frac{2\cot x}{1 + \cot^2 x}$ ($\cot^2 x \to \frac{1}{\sin^2 x}$)

$2\frac{\cos x}{\cancel{\sin x}} \times \sin^{\cancel{2}} x \Rightarrow 2\cos x \sin x$

$2\cos x \sin x = \sin 2x$

$R_y = [-1, 1]$

۹

الف) $y = \sin^6 x + \cos^6 x$

$\sin^6 x + \cos^6 x - \sin^2 x \times \cos^2 x$

$\sin^2 x \to \sin^2 2x \frac{1}{4} \Rightarrow \sin^2 2x \frac{3}{4} - 1$

$0 \leq \quad \leq 1$

$\frac{1}{4} \leq \sin^6 x + \cos^6 x \leq 1$

$R_y = [\frac{1}{4}, 1]$

ب) $y = [\sin^8 x + \cos^8 x] \Rightarrow \{0, 1$

$(\cos^4 x + \sin^4 x)^2 - 2(\sin^4 x \cos^4 x)$

$\sin^4 x \cos^4 x 2 - 1 = \cos^4 + \sin^2 x$

$\sin^4 2x \frac{1}{16} = (\sin^2 2x \frac{1}{4})^2 = \sin^4 x \cos^4 x$

$\sin^4 2x \frac{1}{16} + \sin^2 2x - 1 =$

$0 \leq \sin^2 x \leq 1 \Rightarrow x = 0 \Rightarrow y = 1$; $x = 1 \Rightarrow \frac{1}{8} = y$

$\frac{1}{8} \leq \sin^8 x + \cos^8 \leq 1$

$R_y = \{0, 1\}$

۱۰

الف) $y = \frac{2x^2 + 7x - 4}{x + 4}$

$2x^2 + 7x - 4 \,|\, x + 4$; $2x^2 + 8x$; $2x - 1$; $-8x - 4$; $8x + 4$; 0

$\frac{(2x-1)\cancel{(x+4)}}{\cancel{x+4}}$

$y = 2x - 1$

$x \neq -4$

$y \neq -9$

$R_y = R - \{-9\}$

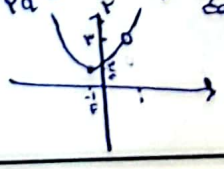
ب) $y = \frac{x^3 - 1}{x - 1}$

$x \neq 1$

$y \neq 3$

$\frac{\cancel{(x-1)}(x^2 + x + 1)}{\cancel{x-1}} \Rightarrow x^2 + x + 1$

$\frac{-b}{2a} = -\frac{1}{2} \Rightarrow \frac{-1}{2a} = \frac{3}{4}$



$R_y = [\frac{3}{4}, +\infty) - \{3\}$