پاسخنامه تشریحی تکلیف ۱ — یازدهم B — دبیر ریاضی: بنیامین تن فر

۱)

$f(x)=\sqrt{1-x^2}$

$g=\{(-1,1)(0,4)(2,0)(1,2)\} \to 2g=\{(-1,2)(0,8)(2,0)(1,4)\}$

$D_f = 1-x^2 \geq 0 \Rightarrow (1-x)(1+x)\geq 0$

$\Rightarrow D_f=[-1,1]$

$2g-3f=?$ برد

$$\begin{cases} f(-1)=\sqrt{1-1}=\boxed{0} \xrightarrow{\times 3} =\boxed{0} \\ f(0)=\sqrt{1}=\boxed{1} \xrightarrow{\times 3} =\boxed{3} \\ f(1)=\boxed{0} \xrightarrow{\times 3} =\boxed{0} \end{cases}$$

$\Rightarrow$ برد $2g-3f=\{2,5,4\} \leftarrow$ جواب نهایی

۲)

$$\begin{cases} f(x)=2x-1 \\ D_f=[3,+\infty) \end{cases} \quad R_f \Rightarrow 2(3)-1=5 \to [5,+\infty)$$

$$\begin{cases} g(x)=\frac{1}{3}x+3 \\ D_g=(-\infty,3] \end{cases} \quad R_g \Rightarrow \frac{1}{3}(3)+3 \to (-\infty,4]$$

$R_f\cup R_g=\boxed{\mathbb{R}-(4,5)}$

۳)

$y=\frac{-x^2}{2}+x+3 \longrightarrow \frac{-x^2}{2}+x+3>\frac{3}{2}$

$\longrightarrow -x^2+2x+6>3 \Rightarrow 0>x^2-2x-3$

$\sqrt{b-a}=\boxed{2}$

$$\begin{array}{c|c|c} & -1 & 3 \\ + & - & + \end{array}$$

$(-1,3)$: $a=-1$, $b=3$

۴)

$y=|x-1|+|x-3|+|2x-4|$

1 3 2

$$y\begin{cases} -4x+8 & x<1 \\ -2x+6 & 1\leq x<2 \\ 2x-2 & 2\leq x<3 \\ 4x-8 & x\geq 3 \end{cases}$$

$\Rightarrow x=1 \rightsquigarrow -2(1)+6=\boxed{4}$

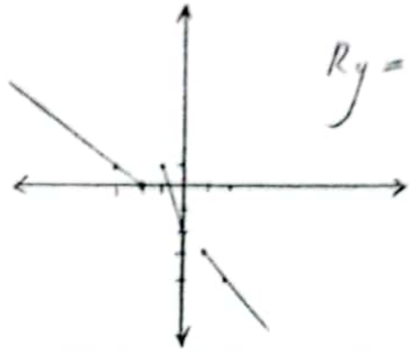
$\Rightarrow x=2 \rightsquigarrow 2(2)-2=\boxed{2}$ جواب ✓

$\Rightarrow x=3 \rightsquigarrow 4(3)-8=\boxed{4}$

۵)

$$y = |x| - 2|x+1|$$

$$y = \begin{cases} x+2 & x<-1 \\ -3x-2 & -1 \leqslant x < 0 \\ -x-2 & x \geqslant 0 \end{cases}$$



$$R_y = \mathbb{R} - (-3,-2)$$

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۶)

$$y = \frac{x^2+5x+m}{x+1} \qquad R_y = \mathbb{R} \qquad yx + y = x^2+5x+m \qquad x^2+5x+m-yx-y=0$$

$$x^2 + x(5-y) + (m-y) = 0 \Rightarrow x = \frac{y-5 \pm \sqrt{(5-y)^2 - 4(1)(m-y)}}{2} \qquad \Delta \geqslant 0$$

$$25 - 10y + y^2 - 4m + 4y \geqslant 0 \Rightarrow y^2 - 6y + 25 - 4m \geqslant 0 \qquad \Delta \leqslant 0$$

اگر بخواهد این عبارت همواره + باشد باید این سهمی همواره بالای x یا مماس بر آن باشد

$$\Delta_y : \underbrace{(-6)^2}_{36} \underbrace{- 4(1)(25-4m)}_{-100 \; + 16m} \leqslant 0 \Rightarrow 16m \leqslant 64 \Rightarrow m \leqslant 4 \quad \underbrace{\{1,2,3,4\}}_{\text{جواب}}$$

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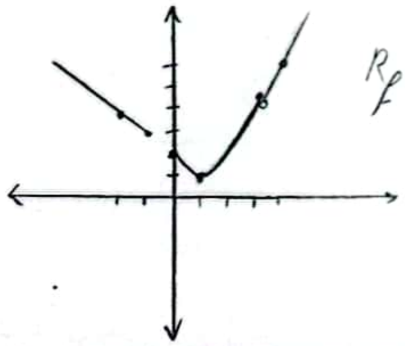
۷)

$$f(x) = \begin{cases} x+2 & x \geqslant 3 \\ x^2-2x+2 & 0 \leqslant x < 3 \\ |x|+2 & x<0 \end{cases}$$



$$\boxed{R_f = [1,+\infty)}$$

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۸)

$$f(x) = \begin{cases} x^2+4x+3 & x \leqslant 0 \\ [2x]-2x & x>0 \end{cases} \longrightarrow \begin{cases} x = -\frac{b}{2a} = -\frac{4}{2} = -2 \\ y = (-2)^2 + 4(-2) + 3 = -1 \end{cases}$$

$$(I)\ x \leqslant 0 \ , a=1 \qquad (II)\ x>0 \Rightarrow f(x) = [2x]-2x \Rightarrow [2x] \leqslant 2x < [2x]+1$$

$$[2x]-2x \in \underbrace{(-1,0]}_{(II)} \ \Big/ \ f(0) = 0^2+0+3 = 3 \longrightarrow R_y = \underbrace{[-1,3]}_{(I)}$$

$$I \cup II \Rightarrow (-1,0) \cup [-1,3] = [-1,3] \longleftarrow \text{جواب}$$

(9

$y = a+1-\sqrt{2x+3} \quad D_y=[b,+\infty) \quad R_y=(-\infty,5] \quad ab=?$

$2x+3 \geq 0 \Rightarrow 2x \geq -3 \Rightarrow x \geq -\frac{3}{2} \rightsquigarrow \left[-\frac{3}{2},+\infty\right) \longrightarrow \underline{\underline{b}}$

برد: $\sqrt{2x+3} \geq 0 \xrightarrow[\times(-1)]{} -\sqrt{2x+3} \leq 0 \Rightarrow \underbrace{a+1-\sqrt{2x+3}}_{y} \leq a+1$

افزودن a+1 به طرفین نامساوی

$y \leq a+1 \rightsquigarrow R_y=(-\infty,a+1] \quad a+1=5 \Rightarrow \boxed{a=4}$

$ab = 4 \times -\frac{3}{2} = \boxed{-6}$ جواب

(10.

$f(x) = 2\sqrt{x+1}+4\sqrt{1-x} \qquad y=\frac{f(x)}{6}-\sqrt{2+2\sqrt{1-x^2}}+\frac{f(x)}{3}$

$f(x)+g(x)=3\sqrt{2+2\sqrt{1-x^2}} \qquad g(x)=3\sqrt{2+2\sqrt{1-x^2}}-f(x)$

$D_f=D_g \qquad y=\frac{f(x)}{6}-\frac{g(x)}{3}=\frac{1}{2}f(x)-\sqrt{2+2\sqrt{1-x^2}}$

$\Rightarrow \frac{1}{2}\left(2\sqrt{x+1}+4\sqrt{1-x}\right)-\sqrt{2+2\sqrt{1-x^2}}$

$y=\sqrt{x+1}+2\sqrt{1-x}-\sqrt{2+2\sqrt{1-x^2}} \qquad -1 \leq x \leq 1$

$x=1 \xrightarrow{y} 0$

$x=-1 \longrightarrow \sqrt{2}$

$x=0 \longrightarrow 1$

$\boxed{R=[0,\sqrt{2}]}$