

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & ; x \leq 1 \\ \sqrt{x^2+1} & ; x > 1 \end{cases}$$

$$f\left(\frac{2}{3}\right) = \cot \frac{\pi}{4} = \sqrt{3}$$

$$f\left(f\left(\frac{2}{3}\right)\right) = f(\sqrt{3}) = 2$$

$\hookrightarrow f \circ f\left(\frac{2}{3}\right)$

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$$g\left(\frac{\pi}{3}\right) = 2 \times \underbrace{\cos^2 \frac{\pi}{3}}_{\frac{1}{4} \times \frac{1}{4}} = \frac{1}{2}$$

$$f \circ g\left(\frac{\pi}{3}\right) = f\left(\frac{1}{2}\right) \xrightarrow{x=2} \sqrt{\frac{4-1}{2}} = \left(\frac{\sqrt{3}}{2}\right)$$

الف) $f \circ g = \left[\frac{x}{1-x} \right] \rightarrow \left[\frac{x+1-2}{1-x} \right] = \left[\frac{1}{x-1} - 1 \right]$
 $= \left[\frac{1}{x-1} \right] - 1$
 ب) $f \circ g(\sqrt{2}) = \left[\frac{1}{\sqrt{2}-1} \right] - 1 = \left[\frac{\sqrt{2}+1}{1} \right] - 1 = 2-1 = 1$

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$$f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = \frac{\sqrt{2}}{2}$$

$$g \circ f\left(\frac{\pi}{2}\right) = g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$$

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الف) $f \circ g(x) = \{(2, 5), (4, 2), (1, 2)\}$

ب) $g \circ f(x) = \emptyset$

ج) $f \circ f(x) = \emptyset$

د) $g \circ g(x) = \{(2, 4), (4, 1)\}$

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$$(2, 4) \in f \circ g$$

$$\hookrightarrow a = 2$$

$$(2, 4) \in g \circ f$$

$$\hookrightarrow b = 4$$

$$(a, b) = (2, 4)$$

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$f(f(x)) = a(ax+b)+b = a^2x+ab+b = \varepsilon x + \gamma$ $a^2 = \varepsilon \rightarrow a = \pm \sqrt{\varepsilon}$ $\begin{cases} a = \sqrt{\varepsilon} \rightarrow b = 1 \rightarrow f(x) = \sqrt{\varepsilon}x + 1 \\ a = -\sqrt{\varepsilon} \rightarrow b = -\gamma \rightarrow f(x) = -\sqrt{\varepsilon}x - \gamma \Rightarrow f(-1) = -1 \end{cases}$ $g(-1) = g(\sqrt{\varepsilon}x + \gamma)$ $x = -\sqrt{\varepsilon} \Rightarrow \gamma(-\sqrt{\varepsilon}) - \gamma = -1$	6
$\begin{aligned} x \in D_f &\rightarrow \mathbb{R} \\ f(x) \in D_g &\rightarrow \mathbb{R}^+ - \{1\} \\ &\hookrightarrow \mathbb{R} - \{0, \varepsilon\} \end{aligned} \quad \left. \begin{aligned} &\mathbb{R}^+ - \{1\} \\ &x+ x \geq 0 \\ &x \in \mathbb{R} \end{aligned} \right\}$ $x+ x \neq 0, \quad x+ x \neq 1$ $x \neq 0 \text{ متقارباً, } x \neq 1$	7
$D_f = [-1, 1], \quad D_g = [0, +\infty) \xRightarrow{\cap} D(f+g) = [0, 1]$ $\begin{cases} x \in D_f \rightarrow [-1, 1] \\ f(x) \in D(f+g) \rightarrow \underbrace{0 \leq \sqrt{1-x^2}}_{\text{محمول}} \leq 1 \rightarrow 1-x^2 \leq 1 \rightarrow x^2 \geq 0 \checkmark \end{cases}$ $\boxed{[-1, 1]}$	8
$\text{الف) } f\left(\frac{t^3x+1}{x-2}\right) = \varepsilon x + \delta$ $3x+1 = t^3x-2t \rightarrow x = \frac{1+2t}{t-3} \Rightarrow \varepsilon x + \delta = \frac{13t-7}{t-3}$ $\text{ب) } f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$ $t^3 = x^3 + \frac{1}{x^3} + 2x + \frac{2}{x} \Rightarrow x^3 + \frac{1}{x^3} = t^3 - 2t \quad \left. \begin{aligned} &t^3 = x^3 + \frac{1}{x^3} + 2x + \frac{2}{x} \\ &t^3 - 2t = x^3 + \frac{1}{x^3} \end{aligned} \right\} f(x) = x^3 - 2x$	9
$\begin{aligned} g(x) &= a(x-1)(x-2\sqrt{2}) \\ f(x) &= x\sqrt{x} \end{aligned} \quad \left. \begin{aligned} &g \circ f(x) = a(x\sqrt{x}-1)(x\sqrt{x}-2\sqrt{2}) \\ &x=1, 2 \text{ نقاطی که محور } x \text{ را قطع می کنند (ریشه ها)} \\ &x=1, 2 \end{aligned} \right\}$ $\text{اختلاف} = 2-1 = \boxed{1}$	10