

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & x < 1 \\ \sqrt{x^2 + 1} & x > 1 \end{cases} \quad f \circ f\left(\frac{1}{2}\right) = ? \quad \frac{1}{2} < 1$$

$$\textcircled{1} \rightarrow \cot \frac{\frac{1}{2} \pi}{4} = \cot \frac{\pi}{8} = \sqrt{2}$$

$$\textcircled{2} \rightarrow \sqrt{2+1} = \sqrt{3} = \textcircled{2}$$

$$\frac{\pi}{4} \xrightarrow{f} \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$f\left(\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{1}{2} - 1} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

$$f \circ g\left(\frac{\pi}{4}\right) \quad g(x) = 2 \cos x \quad f(x) = \sqrt{\frac{x-1}{2x}}$$

$$\sqrt{2} \xrightarrow{g} \frac{\sqrt{2}}{1-\sqrt{2}}$$

$$\frac{\sqrt{2}}{1-\sqrt{2}} \xrightarrow{f} \left[\frac{\sqrt{2}}{1-\sqrt{2}} \right] \approx [-1, \infty) = [-1]$$

$$f \circ g\left(\frac{\pi}{4}\right) \quad g(x) = \frac{x}{1-x} \quad f(x) = [x]$$

$$\frac{\pi}{4} \xrightarrow{f} \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\frac{\sqrt{2}}{2} \xrightarrow{g} \frac{\frac{\sqrt{2}}{2}}{1-\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\frac{2-\sqrt{2}}{2}} = \frac{1}{\sqrt{2}}$$

$$g \circ f\left(\frac{\pi}{4}\right) = ? \quad g(x) = x\sqrt{1-x^2} \quad f(x) = \sin x$$

$$\Rightarrow g \circ f(x)$$

$$\begin{array}{l} 1 \rightarrow 2 \times \\ 2 \rightarrow 1 \times \\ 3 \rightarrow 4 \times \\ 4 \rightarrow 3 \times \end{array} \quad \emptyset$$

$$\Rightarrow g \circ g(x)$$

$$\begin{array}{l} 1 \rightarrow 2 \rightarrow 1 \\ 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 4 \rightarrow 3 \\ 4 \rightarrow 3 \rightarrow 4 \end{array}$$

$$\{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$f(x) = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$g(x) = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$\text{الف} \quad f \circ g(x) =$$

$$\{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$\begin{array}{l} 1 \rightarrow 2 \rightarrow 1 \\ 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 4 \rightarrow 3 \\ 4 \rightarrow 3 \rightarrow 4 \end{array}$$

$$\begin{array}{l} 1 \rightarrow 2 \times \\ 2 \rightarrow 1 \times \\ 3 \rightarrow 4 \times \\ 4 \rightarrow 3 \times \end{array} \quad \emptyset$$

$$f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$g = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$(1, 2) \in f \circ g$$

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$$(a, b) = ?$$

$$\text{اگر } a = 1 \text{ باشد}$$

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$$(1, 2)$$

$$a = 1$$

$$b = 2$$

f و g خطی $f(f(x)) = 7x + 3$ $f(f(x)) = a(ax+b)+b = 7x+3$ $a^2x + ab + b = 7x + 3$ $a^2 = 7 \rightarrow a = \sqrt{7}$ $2b + b = 3 \rightarrow b = 1$ $f(x) = \sqrt{7}x + 1$	$g(7x+3) = 3x-2$ $g(x) = cx+d \rightarrow c(7x+3)+d = 3x-2$ $7cx + 3c + d = 3x - 2$ $7c = 3 \rightarrow c = \frac{3}{7}$ $3 \times \frac{3}{7} + d = -2$ $d = -2 - \frac{9}{7} = -\frac{14}{7} - \frac{9}{7} = -\frac{23}{7}$ $g(x) = \frac{3}{7}x - \frac{23}{7}$ $g \circ f(-1) = g(f(-1)) \Rightarrow \frac{3}{7}(7(-1)+3) - \frac{23}{7} = \frac{3}{7}(-4) - \frac{23}{7} = -\frac{12}{7} - \frac{23}{7} = -\frac{35}{7} = -5$ $g \circ f(x) = \frac{3x-10}{7} \xrightarrow{x=-1} = \frac{-3-10}{7} = \frac{-13}{7}$	$g \circ f(-1) = ?$
$f(x) = \sqrt{x+1}$ $g(x) = \frac{1}{x^2-1}$ $Dg \circ f = ?$ $g \circ f = g(f(x))$ $x \in Df \rightarrow x+1 \geq 0 \rightarrow [0, +\infty)$ $Dg \rightarrow x^2-1 \neq 0$ $x(x-1) \neq 0$ $x \neq 0, 1$ $x+1 \neq 1$ $x \neq 0$ $x \neq 1$ $Dg \circ f = (0, +\infty) - \{1\}$	$f(x) \in Dg$ $x > 0$ $x \in (0, +\infty)$	
$f(x) = \sqrt{1-x^2}$ $g(x) = \sqrt{x}$ $D(f+g) \circ f = ?$ $(f+g) \circ f \rightarrow f \circ f(x) + g \circ f(x) =$ $f \circ f(x) \rightarrow x \in Df \rightarrow Df \rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1$ $f(x) \in Dg \rightarrow f(x) \in Dg \rightarrow -1 \leq \sqrt{1-x^2} \leq 1 \rightarrow 0 \leq 1-x^2 \leq 1 \rightarrow -1 \leq x^2 \leq 0 \rightarrow 0 \leq x^2 \leq 1 \rightarrow -1 \leq x \leq 1$ $g \circ f(x) \rightarrow x \in Df \rightarrow -1 \leq x \leq 1$ $f(x) \in Dg \rightarrow \sqrt{1-x^2} \geq 0 \rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1$ $D(f+g) \circ f \rightarrow [-1, 1]$ $Dg \rightarrow x \geq 0$		
$f(x) = \frac{x^2+1}{x-2}$ $f(t) = \frac{t^2+1}{t-2} + \omega = \frac{-1t-1}{t-2} + \omega = \frac{-1t-1+t\omega-\omega t}{t-2} = \frac{-1^2t+11}{t-2} \Rightarrow f(x) = \frac{-1^2x+11}{x-2}$ $x = \frac{-1t-1}{t-2}$ $f(x) = \frac{x^2+1}{x-2}$ $x + \frac{1}{x} = t \rightarrow (x + \frac{1}{x})^2 = x^2 + 2 + \frac{1}{x^2} \Rightarrow x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2 = t^2 - 2 \Rightarrow f(t) = t^2 - 2 \Rightarrow f(x) = x^2 - 2$	$f(x) = ?$	
$g \circ f(x) = g(f(x))$ $if g(f(x))$, $f(x) = 1 \rightarrow g(f(x)) = 0 \rightarrow if f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$ $if g(f(x))$, $f(x) = \sqrt{2} \Rightarrow g(f(x)) = 0 \rightarrow if f(x) = \sqrt{2} \Rightarrow x\sqrt{x} = \sqrt{2} \Rightarrow x = 2$ $x-1 = 1$		