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مسئله ریاضی

9) الف) $f\left(\frac{3x+1}{x+2}\right) = 4x + 5$

$$\frac{3x+1}{x+2} = t \Rightarrow 3x+1 = t(x+2)$$

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$$x(3-t) = -2t-1 \Rightarrow x = \frac{-(2t+1)}{3-t} = \frac{2t+1}{t-3}$$

$$f\left(\frac{2t+1}{t-3}\right) + 5 = \frac{12t+4}{t-3} + \frac{5t-15}{t-3} = \frac{17t-11}{t-3} = \frac{17x-11}{x-3} = f(x)$$

ب) $f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x + \frac{3}{x}$$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = x^3 - 3x = f(x)$$

10) $g(1) = 0$, $g(\sqrt{x}) = 0$

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$$(g \circ f)(x) = 0 \Rightarrow g(f(x)) = 0$$

$$f(x) = 1 \quad \text{و} \quad f(x) = \sqrt{x} \quad \quad f(x) = x\sqrt{x} = x \times x^{\frac{1}{2}} = x^{\frac{3}{2}}$$

$$x^{\frac{3}{2}} = 1 \Rightarrow x = 1 \quad \quad x^{\frac{3}{2}} = \sqrt{x} = 2 \times 2^{\frac{1}{2}} = 2^{\frac{3}{2}} \quad x = 2$$

$$2-1 = 1$$

$$2) F \circ g \left(\frac{1}{\sqrt{p}} \right) = F \left(\frac{1}{\sqrt{p}} \right) = \sqrt{\frac{p^2 x^2 - 1}{x^2}} \Rightarrow x = \frac{1}{\sqrt{p}} \Rightarrow \sqrt{\frac{p^2 - 1}{p}} = \sqrt{\frac{p}{p}} \quad (\text{اف})$$

$$\cos \left(\frac{1}{\sqrt{p}} \right) = \frac{1}{\sqrt{p}} \quad \frac{1}{x} = \frac{1}{\sqrt{p}}$$

$$g(x) = \frac{1}{x} \left(\frac{1}{\sqrt{p}} \right)^2 = \frac{1}{p}$$

$$F \circ g(\sqrt{p}) \quad \left[\frac{x}{1-x} \right] \Rightarrow \left[\frac{\sqrt{p}}{1-\sqrt{p}} \right] \stackrel{\text{نوع 1}}{=} [-1-\sqrt{p}] = -1-p \quad (\text{ب})$$

$$3) F(x) = \sin x \quad g(x) = x \sqrt{1-x^2} \quad g \circ F \left(\frac{1}{\sqrt{p}} \right) = \frac{1}{\sqrt{p}}$$

$$g(F(x)) = \sin x \sqrt{1-\sin^2 x} = \sin x \cos x \Rightarrow x = \frac{1}{\sqrt{p}} \quad (\text{ج})$$

$$\frac{\sqrt{p}}{p} \times \frac{\sqrt{p}}{p} = \left(\frac{1}{p} \right)$$

$$4) F(F(x)) = px + q \Rightarrow F(ax+b) \Rightarrow a(ax+b) + b = a^2 x + ab + b = px + q$$

$$a = p \Rightarrow px + 1$$

$$b = 1$$

$$g(px+q) = px - 1$$

$$g(px+q) + b = px - 1 \Rightarrow pa + q + b = px - 1 \Rightarrow a = \frac{p}{p} = 1$$

$$q = 1 \cdot a - 1, a \quad b = -1, a$$

$$g(F(-1)) = -1$$

$$1) F(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad D(F+g) \text{ or } F \circ F + g \circ F \Rightarrow F(F(x)) + g(F(x)) \Rightarrow \sqrt{\sqrt{1-x^2}} \quad (1)$$

$$F(x) = \sqrt{1-x^2} \geq 0 \quad \Downarrow$$

$$D_F = 1-x^2 \geq 0 \quad \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{1-(1-x^2)} = \sqrt{x^2} = |x|$$

$$-1 \leq x \leq 1$$

$$D = [-1, 1]$$

$$D_f = -1 \leq x \leq 1 \quad \cap \quad D_g = x \geq 0 \rightarrow -1 \leq x \leq 1 \quad \leftarrow \sqrt{1-x^2} \leq 1 \rightarrow x^2 \geq 0 \quad \checkmark$$

$$D(F+g) = [-1, 1]$$