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$$f(n) = \begin{cases} \cot \frac{\pi n}{2} & ; n \leq 1 \\ \sqrt{n^2 + 1} & ; n > 1 \end{cases} \quad f(f(\frac{1}{2})) \Rightarrow f(\frac{1}{2}) : \cot \frac{\pi}{4} = \cot \frac{\pi}{4} = 1$$

$$f(\sqrt{2}) : \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3}$$

$$\text{اذا } f(\frac{1}{n}) = \sqrt{\frac{n^2 - 1}{n^2}} \quad , \quad g(n) = 2 \cos^2 n \quad , \quad f \circ g(\frac{\pi}{4})$$

$$f(\frac{1}{n}) = \sqrt{\frac{n^2 - 1}{n^2}} \rightarrow \frac{1}{n} = t \quad ; \quad \frac{n^2 - 1}{n^2} = \frac{n^2}{n^2} - \frac{1}{n^2} = \frac{n^2}{n^2} - \frac{1}{n^2} = 1 - t^2$$

$$\Rightarrow f(t) = \sqrt{1 - t^2} \Rightarrow f(n) = \sqrt{1 - \frac{1}{n^2}}$$

$$f(g(\frac{\pi}{4})) \Rightarrow g(n) = 2 \cos^2 n = \frac{\pi}{4} \quad , \quad 2 \times \frac{1}{2} = 1$$

$$\Rightarrow f(\frac{1}{2}) = \sqrt{1 - (\frac{1}{2})^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\hookrightarrow f(n) = [n] \quad , \quad g(n) = \frac{n}{1-n} \quad , \quad f \circ g(\sqrt{2})$$

$$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2} + 2}{-1} = -\sqrt{2} - 2$$

$$f(-\sqrt{2} - 2) \Rightarrow [-\sqrt{2} - 2] = -3, -4, \dots$$

$$f(n) = \sin n \quad , \quad g(n) = n \sqrt{1 - n^2} \quad , \quad g \circ f(\frac{\pi}{2}) = g(f(\frac{\pi}{2})) = ?$$

$$f(\frac{\pi}{2}) = \sin(\frac{\pi}{2}) = 1$$

$$g(\frac{1}{2}) = \frac{1}{2} \sqrt{1 - (\frac{1}{2})^2} = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$$

$$f(n) = \{(1, \omega), (2, \omega), (3, \omega), (4, \omega)\} \quad , \quad g(n) = \{(1, \omega), (2, \omega), (3, \omega), (4, \omega)\}$$

$$\text{اذا } f \circ g(n) : \begin{matrix} 1 & \xrightarrow{g(n)} & \omega & \xrightarrow{f(n)} & \omega \\ 2 & \xrightarrow{g(n)} & \omega & \xrightarrow{f(n)} & \omega \\ 3 & \xrightarrow{g(n)} & \omega & \xrightarrow{f(n)} & \omega \\ 4 & \xrightarrow{g(n)} & \omega & \xrightarrow{f(n)} & \omega \end{matrix}$$

$$\hookrightarrow g \circ f(n) : \begin{matrix} 1 & \xrightarrow{f(n)} & \omega & \xrightarrow{g(n)} & \omega \\ 2 & \xrightarrow{f(n)} & \omega & \xrightarrow{g(n)} & \omega \\ 3 & \xrightarrow{f(n)} & \omega & \xrightarrow{g(n)} & \omega \\ 4 & \xrightarrow{f(n)} & \omega & \xrightarrow{g(n)} & \omega \end{matrix}$$

$$f \circ g(n) = \{(1, \omega), (2, \omega), (3, \omega), (4, \omega)\}$$

$$g \circ f(n) = \emptyset$$

$$2.) f \circ f(n) : \begin{matrix} 1 & \xrightarrow{f(n)} & \omega & \xrightarrow{f(n)} & \omega \\ 2 & \xrightarrow{f(n)} & \omega & \xrightarrow{f(n)} & \omega \\ 3 & \xrightarrow{f(n)} & \omega & \xrightarrow{f(n)} & \omega \\ 4 & \xrightarrow{f(n)} & \omega & \xrightarrow{f(n)} & \omega \end{matrix}$$

$$f \circ f(n) = \emptyset$$

$$\hookrightarrow g \circ g(n) : \begin{matrix} 1 & \xrightarrow{g(n)} & \omega & \xrightarrow{g(n)} & \omega \\ 2 & \xrightarrow{g(n)} & \omega & \xrightarrow{g(n)} & \omega \\ 3 & \xrightarrow{g(n)} & \omega & \xrightarrow{g(n)} & \omega \\ 4 & \xrightarrow{g(n)} & \omega & \xrightarrow{g(n)} & \omega \end{matrix}$$

$$g \circ g(n) = \{(1, \omega), (2, \omega), (3, \omega), (4, \omega)\}$$

$$f = \{(1, \omega), (2, \omega), (3, \omega), (4, \omega)\} \quad , \quad g = \{(1, \omega), (2, \omega), (3, \omega), (4, \omega)\} \quad , \quad (1, \omega) \in f \quad , \quad (1, \omega) \in g \quad , \quad (a, b) = ?$$

$$f(g(1)) = 2 \Rightarrow \boxed{1} \xrightarrow{g(n)} 2 \xrightarrow{f(n)} 2 \Rightarrow \boxed{2} = a = 2$$

$$g(f(1)) = 1 \Rightarrow 1 \xrightarrow{f(n)} \omega \xrightarrow{g(n)} 1 \Rightarrow \boxed{1} = a = 1$$

$$(a, b) \Rightarrow (1, \omega)$$

$$\boxed{b = \omega} \quad , \quad \omega = \text{نقطه اول و دوم}$$

$$f(f(x)) = \varepsilon x + r \quad g(x+r) = rx - r \quad g \circ f(-1) = ?$$

$$\hookrightarrow f(x) = ax + b \quad \hookrightarrow g(x) = cx + d$$

$$f(f(x)) = a(ax+b)+b = a^2x+ab+b$$

$$a^2x+ab+b = \varepsilon x + r \rightarrow \begin{cases} a^2 = \varepsilon \rightarrow a = \pm \sqrt{\varepsilon} \\ ab+b = r \xrightarrow{a=\sqrt{\varepsilon}} b \neq 1 : f(x) = \sqrt{\varepsilon}x + 1 \end{cases}$$

$$g(rx+r) = rx - r$$

$$c(rx+r)+d = rx - r \rightarrow \begin{cases} rc = r \rightarrow c = \frac{r}{r} \\ rc+d = -r : r(\frac{r}{r})+d = -r \rightarrow d = -\frac{1r}{r} \end{cases}$$

$$g \circ f(-1) \Rightarrow f(x) : rx+1 \rightarrow f(-1) = r(-1)+1 = -1$$

$$g(-1) = \frac{r}{r}(-1) - \frac{1r}{r} = -\frac{1r}{r} = -1$$

$$g \circ f(-1) \Rightarrow f(x) = -rx - r \rightarrow f(-1) = -r(-1) - r = -1$$

$$g \circ f(-1) = -1$$

$$g(-1) = \frac{r}{r}(-1) - \frac{1r}{r} = -1$$

$$f(x) = \sqrt{x+1} \quad g(x) = \frac{1}{x^2 - \varepsilon x} \quad \cap g \circ f : \{x \in D_f : f(x) \in D_g\}$$

$$\sqrt{x+1} \rightarrow x+1 \geq 0 \Rightarrow \begin{cases} 0 & ; x < 0 \\ \sqrt{x} & ; x \geq 0 \end{cases} \Rightarrow D_f = \mathbb{R}$$

$$g(x) = \frac{1}{x^2 - \varepsilon x} \Rightarrow x^2 - \varepsilon x \neq 0 \rightarrow x(x - \varepsilon) \neq 0 \quad x \neq 0, \varepsilon$$

$$\{x \in \mathbb{R} \cap \sqrt{x+1} \neq \{0, \varepsilon\}\}$$

$$\hookrightarrow \sqrt{x+1} = 0 \rightarrow x \leq 0 \quad \text{بزرگتر از صفر}$$

$$\hookrightarrow \sqrt{x+1} = \varepsilon \rightarrow \sqrt{x} = \varepsilon \rightarrow x = \varepsilon^2 \rightarrow x \neq 1$$

$$\Rightarrow D_{g \circ f} = (0, +\infty) - \{1\} \subseteq (0, 1) \cup (1, +\infty)$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad D_{f+g} \circ f$$

$$D_f \Rightarrow 1-x^2 \geq 0 \rightarrow x^2 \leq 1 \rightarrow x \in [-1, 1]$$

$$D_g \Rightarrow \sqrt{x} \rightarrow x \geq 0 \rightarrow x \in [0, +\infty)$$

$$D_{f+g} = D_f \cap D_g \rightarrow [-1, 1] \cap [0, +\infty) = [0, 1]$$

$$D_{(f+g) \circ f} = \{x \in D_f : f(x) \in D_{f+g}\} = \{-1 \leq x_1 : 0 \leq \sqrt{1-x_1^2} \leq 1\} \Rightarrow D_{(f+g) \circ f} = [-1, 1]$$

$$\text{برای } [-1, 0] \text{ و } [0, 1] \text{ داریم: } \sqrt{1-x^2} \text{ و } \sqrt{x} \text{ را می بینیم}$$

$$\text{الف) } f\left(\frac{rx+1}{x-r}\right) = \varepsilon x + \omega$$

$$\frac{rx+1}{x-r} = t \rightarrow rx+1 = tx - rt \rightarrow rt+1 = tx - rx \Rightarrow rt+1 = x(t-r) \Rightarrow x = \frac{rt+1}{t-r}$$

$$\rightarrow \varepsilon\left(\frac{rt+1}{t-r}\right) + \omega = \frac{\varepsilon rt + \varepsilon}{t-r} + \omega = \frac{\varepsilon rt + \varepsilon + \omega t - \omega}{t-r} = \frac{(\varepsilon r + \omega)t - (\omega - \varepsilon)}{t-r} \Rightarrow \frac{p}{f(x)} = \frac{1rx - 11}{x-r}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = rx + \frac{1}{x^r}$$

$$x + \frac{1}{x} = t \quad ; \quad rx + \frac{1}{x^r} = \left(x + \frac{1}{x}\right)^r - \frac{1}{x^r} = t^r - \frac{1}{x^r} = t^r - rt$$

$$f(t) = t^r - rt \rightarrow f(x) = rx - \frac{1}{x^r}$$

$$g(1) = 0, g(2\sqrt{2}) = 0$$

$$g(\underline{f(n)}) = 0 \rightarrow f(n) = 1 \text{ و } f(n) = 2\sqrt{2}$$

$$f(n) = n\sqrt{n} = 1 \rightarrow n = 1$$

$$f(n) = n\sqrt{n} = 2\sqrt{2} \rightarrow n = 2$$

بنابراین $g \circ f(n)$ در $n = 1, 2$ محور n ها را قطع می کند

$$2 - 1 = 1$$

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