نام و نام خانوادگی: دیبا رستم‌زاده

پاسخنامه تشریحی تکلیف شماره ...... کلاس یازدهم [illegible]

۱

$f(f(x)) \longrightarrow f(f(\frac{2}{3})) \longrightarrow f(\frac{2}{3}) \rightarrow \cot\frac{2\pi}{12} = \cot\frac{\pi}{6} = \sqrt{3}$

$f(\sqrt{3}) \longrightarrow \sqrt{(\sqrt{3})^2+1} \longrightarrow \sqrt{4} = 2$

$fof(\frac{2}{3}) = 2$

۲

الف) $x = \frac{1}{t} = \frac{\frac{2}{t}-1}{\frac{1}{t}+2} = \frac{\frac{2-t}{t}}{\frac{1}{t}+2} = \quad 2t^2 - t^3 = t \quad t^2(2-t) = t \quad t^2 = \frac{t}{2-t} \quad t = \pm\sqrt{\frac{t}{2-t}}$

$x = \pm\sqrt{\frac{x}{2-x}} \longrightarrow \neq$

$fog(\frac{\pi}{3}) = \sqrt{\frac{2\cos^2 x}{2-2\cos^2 x}} \longrightarrow \frac{2(\frac{\sqrt{3}}{2})^2}{2-2(\frac{\sqrt{3}}{2})^2} = \boxed{\sqrt{3} \,,\, -\sqrt{3}}$

ب) $fog(\sqrt{2}) = \left[\frac{x}{1-x}\right] \longrightarrow \left[\frac{\sqrt{2}}{1-\sqrt{2}}\right] \longrightarrow \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{-1} \longrightarrow \left[\frac{\sqrt{2}+2}{-1}\right]$

$\sqrt{2} = 1.4 \quad [-3.4] = -4$

۳

$\frac{\pi}{4} = 45^\circ \qquad \sin 45 = \frac{\sqrt{2}}{2}$

$g(f(x)) = \sin x\sqrt{1-\sin^2 x} \longrightarrow \frac{\sqrt{2}}{2}\sqrt{1-(\frac{\sqrt{2}}{2})^2} \longrightarrow \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}}$

$\frac{\sqrt{2}}{2}\sqrt{\frac{1}{2}} \longrightarrow \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{2}{4} = \frac{1}{2}$

$\downarrow$

$\frac{\sqrt{2}}{2}$

۴

الف) $fog(x) = \{(4,5)(2,5)(8,12)(6,12)\}$

$4 \overset{gx}{=} 5 \overset{fx}{=} 6$

ب) $gof(x) = \varnothing$

ج) $fof(x) = \varnothing$

د) $gog(x) = \{(4,8)(2,6)(6,10)\}$

۵

$fog(x) = \{(1,1)(3,7)(a,2)(b,7)\} \qquad gof(x) = \{(2,2)(3,x)(4,y)(1,z)\}$

$1 \rightarrow 2 \rightarrow 1 \qquad \boxed{a = 4}$

$3 \rightarrow 1 \rightarrow 7$

$a \rightarrow 3 \rightarrow 2$

$b \rightarrow 1 \rightarrow 7$

$(4,2) \in fog \qquad \boxed{(4,5)}$

$2 \rightarrow 1 \rightarrow 2$

$3 \rightarrow 2 \rightarrow x$

$4 \rightarrow 5 \rightarrow y \Longrightarrow 4 \rightarrow 5 \rightarrow 1 \Longrightarrow \boxed{b = 5}$

$1 \rightarrow 7 \rightarrow z$

$(4,1) \in gof$

$f(x) = ax+b$

$f(f(x)) = a(ax+b)+b = 4x+3$

$a^2x+ba+b = 4x+3$

$a^2 = 4 \quad a = \pm 2$

$ba+b = 3 \xrightarrow{a=2} b=1 \Longrightarrow \boxed{2x+1}$

$\xrightarrow{a=-2} b=-3 \Longrightarrow \boxed{-2x-3}$

$2x+3 = t \Longrightarrow x = \frac{t-3}{2}$

$3\left(\frac{t-3}{2}\right) - 2 = \frac{3t-9-4}{2} = \frac{3t-13}{2}$

$\boxed{g(x) = \frac{3x-13}{2}}$

$g(f(-1)) = f(-1) \longrightarrow 2(-1)+1 = -1$

$\longrightarrow -2(-1)+3 = -1$

$g(-1) = \frac{-3-13}{2} = \frac{-16}{2} = \boxed{-8}$ ۶

$D_{gof} = \{x \in D_f \ f(x) \in D_g\} \longrightarrow \{x \in [0,+\infty) , \underbrace{\sqrt{x+|x|} \neq \{0,4\}}\}$

$f(x) = \sqrt{x+|x|} \quad x+|x| \geq 0$

$|x| \geq -x \longrightarrow D_f = [0,+\infty)$

$x \neq$ [illegible] 0

$x \neq 8$

$x^2-4x \neq 0$

$x(x-4) \neq 0$

$x \neq 0 \quad x \neq 4 \longrightarrow D_g = \mathbb{R} - \{0,4\}$

$\boxed{D_{gof} = (0,+\infty) - \{8\}}$ ۷

$f(x) = \sqrt{1-x^2} \quad 1-x^2 \geq 0 \quad x^2 \leq 1 \quad -1 \leq x \leq 1 \quad [-1,1]$

$g(x) = \sqrt{x} \quad [0,+\infty)$

$f+g = D_f \cap D_g \longrightarrow [0,1]$ ۸

الف) $t = \frac{3x+1}{x-2} \longrightarrow tx-2t = 3x+1 \quad x(t-3) = 2t+1 \quad x = \frac{2t+1}{t-3}$

$f(t) = 4t+5 \longrightarrow 4 \times \frac{2t+1}{t-3} + 5 \longrightarrow \frac{8t+4+5t-15}{t-3} \longrightarrow \frac{13t-11}{t-3} \Longrightarrow \boxed{f(x) = \frac{13x-11}{x-3}}$

ب) $x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right) \xrightarrow{t} t^3 - 3t$

$f(t) = t^3 - 3t \quad \boxed{f(x) = x^3 - 3x}$ ۹

$g(1) = 0 \qquad f(x) = x\sqrt{x} = x^{\frac{3}{2}}$

$g(2\sqrt{2}) = 0$

تابع gof محور xها را در ۲ نقطه‌ای ۱ و ۲ قطع می‌کند

$2-1 = 1$

$x^{\frac{3}{2}} = 1 \Longrightarrow x = 1$

$x^{\frac{3}{2}} = 2\sqrt{2} \Longrightarrow x = 2$ ۱۰