۱۹٫۷۵

نام و نام خانوادگی .... پاسخنامه تشریحی تکلیف شماره ۹ کلاس یازدهم ریاضی B ....

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$f(x)=\begin{cases}\cot\frac{\pi x}{4} & x\le 1\\ \sqrt{x^2+1} & x>1\end{cases}$

$f\circ f(\frac{2}{3}) = f(f(\frac{2}{3})) = f(\cot\frac{\pi}{6}) = f(\sqrt{3}) = \sqrt{(\sqrt{3})^2+1} = 2$

(۲)

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الف) $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$ $\quad g(x)=2\cos^2 x$ $\quad f(g(\frac{\pi}{3})) = f(\frac{1}{2}) = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}}$

$g(\frac{\pi}{3}) = \frac{2}{4} = \frac{1}{2}$

ب) $f(x) = [x]$ $\quad g(x)=\frac{x}{1-x}$ $\quad f(g(\sqrt{2})) =$

$g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}\left(\frac{1+\sqrt{2}}{1+\sqrt{2}}\right) = \frac{\sqrt{2}+2}{-1} = -(\sqrt{2}+2)$

$f(-2-\sqrt{2}) = [-\sqrt{2}]-2 = -4$

(۲)

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$f(x)=\sin x$ $\quad g(x)=x\sqrt{1-x^2}$ $\quad g(f(\frac{\pi}{4}))=?$

$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$ $\quad g(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}\sqrt{1-\frac{2}{4}} = \frac{\sqrt{2}}{2}\times\frac{1}{\sqrt{2}} = \frac{1}{2}$

(۲)

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$f(x)=\{(4,5)(6,5)(8,12),(10,12)\}$ $\quad g(x)=\{(4,6)(2,4)(6,8)(8,10)\}$

$f\circ g=\{(4,5),(2,5),(6,12),(8,12)\}$

$g\circ f=\emptyset$

$f\circ f=\emptyset$

$g\circ g=\{(2,6)(4,8)(6,10)\}$

(۲)

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$f=\{(2,1)(3,2),(4,5),(1,7)\}$ $\quad g=\{(1,2)(3,1)(a,3)(b,1)\}$

$f(g(4))=2$ $\quad g(4)=3$ $\quad a=4$

$g(f(4))=1$ $\quad g(5)=1$ $\quad b=5$

$(4,5)$

(۲)

$f(x) = ax+b$

$f(f(x)) = a(ax+b)+b = a^2x+ab+b = 4x+3$ → $a^2=4$ ($a=2$)

↳ $b(a+1)=3$ → $b=1$

$f(x) = 2x+1$

$g(2x+3) = 3x-2$ $\quad 2x+3=t$, $x=\frac{t-3}{2}$

$g(t) = \frac{3t-9}{2} - 2 = \frac{3t-9-4}{2} = \frac{3t-13}{2}$

$g(f(-1)) = g(-1) = \frac{-3-9}{2} - 2 = -8$

(5) ٤

$g(x) = \frac{1}{x^2-4x}$

$x^2-4x = x(x-4)$

	0		4	
	+	0	−	0	+

$D_g = \mathbb{R} - \{0,4\}$

$f(x) = \sqrt{x+|x|}$ $\quad D_{g\circ f} = \{x \in D_f, fx \in D_g\}$

$D_f = \mathbb{R}$

$x \in \mathbb{R}$ ✓ (I), $\sqrt{x+|x|} \neq 0 \rightsquigarrow x \in \mathbb{R}^+$ (II)

$\sqrt{x+|x|} \neq 4 \rightsquigarrow x+|x| \neq 16 \rightsquigarrow x \neq 8$ (III)

$(I) \wedge (II) \wedge (III) = \mathbb{R}^+ - \{8\}$

(5) ٧

$g(x) = \sqrt{x}$ $\quad f(x) = \sqrt{1-x^2}$ $\quad D_{(f+g)\circ f} = \{x \in D_f : (f+g)x \in D_{f+g}\}$

$D_f = \frac{-1 \quad 1}{- \; + \; -}$ $[-1,1]$ $\quad D_g = [0,+\infty)$

$D_{f+g} = [-1,1] \cap [0,+\infty) = [0,1]$ $\quad D_{f+g} = [0,1]$ / $[-1,1]$

$0 \leq \sqrt{1-x^2} \leq 1$

$0 \leq 1-x^2 \leq 1$ → $x^2 \geq 0$ ✓

$-1 \leq -x^2 \leq 0$ $\quad 0 \leq x^2 \leq 1$ $\quad -1 \leq x \leq 1$

(1,V0) ٨

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a) $f\left(\frac{3x+1}{x-2}\right) = 4x+a$

$\frac{3x+1}{x-2} = t$

$x = \frac{2t+1}{t-3}$

$f(t) = \frac{4t+4}{t-3} + a$

$f(x) = \frac{4x+4}{x-3} + a$

$f(x+\frac{1}{x}) = x^3 + \frac{1}{x^3} =$

$(x+\frac{1}{x})^3 - 3(x+\frac{1}{x}) = t^3 - 3t$

$f(x) = x^3 - 3x$

(5) ٩

$g(f(x)) = 1$,

$g(fx) = 2\sqrt{2}$

$xfx = 2\sqrt{2}$ $\quad x=2$

$x\sqrt{x} = 1$ $\quad x=1$

$2-1=1$

(5) ١٠