نام و نام خانوادگی ......[illegible]...... پاسخنامه تشریحی تکلیف شماره ...۹... کلاس ...یازدهم تجربی...

۱

$f(x)=\begin{cases}\cot\frac{\pi x}{6} & ; x\le 1\\ \sqrt{x^2+1} & ; x>1\end{cases}$ $\qquad f\circ f(\frac{2}{3})=?$

$f(f(\frac{2}{3}))$

if $x=\frac{2}{3} \xrightarrow{f(x)} \cot\frac{\pi x}{6} \to \cot\frac{\pi\frac{2}{3}}{24} \to \cot(\frac{\pi}{9}) = 0 \Rightarrow \sqrt{3}$

$f(\sqrt{3})=\sqrt{(\sqrt{3})^2+1}=\sqrt{4}=2$

۲

الف) اگر $f(\frac{1}{x})=\sqrt{\frac{2x-1}{x^2}}$ و $g(x)=2\cos^2x$ مقدار $f\circ g(\frac{\pi}{3})$ را بدست آورید.

$f(\frac{1}{\frac{1}{x}})=f(x)\xrightarrow[\text{if } \frac{1}{x}]{} f(x)=\sqrt{\frac{\frac{2}{x}-1}{\frac{1}{x^2}}}=\sqrt{\frac{\frac{2-x}{x}}{\frac{1}{x^2}}}\Rightarrow\sqrt{x(2-x)}\Rightarrow f(g(x))\to\sqrt{2\underbrace{\cos^2(\frac{\pi}{3})}_{\frac{1}{2}}(\underbrace{2-2\cos^2(\frac{\pi}{3})}_{2-\frac{1}{2}=\frac{3}{2}})}$

$f(g(x))=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$

ب) اگر $f(x)=[x]$ ، $g(x)=\frac{x}{1-x}$ مقدار $f\circ g(\sqrt{2})$ را بدست آورید.

$f(g(x))=[\frac{x}{1-x}]$

if $x=\sqrt{2}$: $[\frac{\sqrt{2}}{1-\sqrt{2}}]=[\frac{\simeq 1.4}{\cancel{1-1.4}\,-0.4}]\simeq[-\frac{7}{2}]=-4$

۳

$g\circ f(\frac{\pi}{4})$ $\qquad g(x)=x\sqrt{1-x^2}$ $\qquad f(x)=\sin x$

$g(f(x))$ $\qquad \sin x\sqrt{\underbrace{1-\sin^2x}_{\cos^2x}}\to g(f(x))=\sin x|\cos x|\xrightarrow[(\cos,\sin>0)]{\frac{\pi}{4}}\sin x\cos x$

$\sin x\cos x\xrightarrow{x=\frac{\pi}{4}}\sin(\frac{\pi}{4})\times\cos(\frac{\pi}{4})=(\frac{\sqrt{2}}{2})^2=\frac{1}{2}$

۴

$g(x)=\{(6,9),(2,4),(9,8),(8,10)\}$ $\qquad f(x)=\{(6,5),(9,5),(8,12),(10,12)\}$

الف) $f\circ g(x)\Rightarrow f(g(x))=\{(4,5),(2,5),(9,12),(8,12)\}$

ب) $g\circ f(x)\Rightarrow g(f(x))=\varnothing$

ج) $f\circ f(x)\Rightarrow f(f(x))=\varnothing$

د) $g\circ g(x)\Rightarrow g(g(x))=\{(4,8),(2,9),(9,10)\}$

۵

$f=\{(2,1),(3,2),(4,5),(1,7)\}$ $\qquad g=\{(1,2),(3,1),(a,3),(b,1)\}$

$(4,2)\in f\circ g$ $\qquad (4,1)\in g\circ f$ $\qquad (a,b)=?$

$f\circ g=f(g(x))$

$g(4)=?$

if $a=4\to g(4)=3\to f(3)=2\to f\circ g=(4,2)\to a=4$

if $b=6\to g(6)=1\to f(1)=7$ ✗

$(4,1)\in g\circ f\to g(f(x))$

$g(f(4))\to g(5)=1$

$b=5$

$\boxed{(a,b)=(4,5)}$

$f(4)=5$

۶

$f(f(x)) = 4x+3$

if $f(x)=ax+b$

$f(f(x)) = a(ax+b)+b = 4x+3$

$a^2x + ab + b = 4x+3$

$a^2 = 4 \Rightarrow a = 2$

$2b+b = 3$

$3b = 3 \Rightarrow b=1 \Rightarrow f(x) = 2x+1$

$g(2x+3) = 3x-2$

$g(x) = cx+d \rightarrow c(2x+3)+d = 3x-2$

$2cx + 3c + d = 3x-2$

$2c = 3 \Rightarrow c = \frac{3}{2} \qquad 3\times\frac{3}{2}+d = -2$

$d = -2 - \frac{9}{2} = -\frac{13}{2}$

$g(x) = \frac{3}{2}x - \frac{13}{2}$

$g\circ f(-1) \rightarrow g(f(-1)) \Rightarrow \frac{3}{2}(2x+1) - \frac{13}{2} = \frac{6x+3-13}{2}$

$g\circ f(x) = \frac{6x-10}{2} \xrightarrow{x=-1} \frac{-16}{2} = -8$

۷

$f(x) = \sqrt{x+|x|} \qquad g(x) = \frac{1}{x^2-4x} \qquad D_{g\circ f} = ?$

$g\circ f \rightarrow g(f(x))$

$x \in D_f \rightarrow \sqrt{x+|x|} \geq 0 \rightarrow [0,+\infty) \quad f(x)\in D_g$

$f(x)\in D_g \qquad \sqrt{x+|x|} \neq 0 \rightarrow$ برای $x\leq 0$: $x+|x| = 0 \Rightarrow x>0$

$\mapsto D_g \rightarrow x^2-4x \neq 0 \qquad x+|x| \neq 0 \rightarrow x\in(0,+\infty)$

$x(x-4) \qquad x+|x| \neq 16$

$x \neq 0, 4 \qquad x \neq 8$

$\boxed{D_{g\circ f} = (0,+\infty) - \{4, 8\}}$

۸

$f(x) = \sqrt{1-x^2} \qquad g(x) = \sqrt{x} \qquad D_{(f+g)\circ f}$

$(f+g)\circ f \rightarrow f\circ f(x) + g\circ f(x) =$

$f\circ f(x) \rightarrow x\in D_f \qquad D_f \rightarrow 1-x^2 \geq 0 \qquad x^2 \leq 1 \rightarrow -1\leq x \leq 1$

$f(x)\in D_f \qquad f(x)\in D_f \quad -1 \leq \sqrt{1-x^2} \leq 1 \rightarrow 0 \leq 1-x^2 \leq 1 \quad -1 \leq -x^2 \leq 0 \quad 1 \geq x^2 \geq 0$

$-1 \leq x \leq 1$

$g\circ f(x) \qquad x\in D_f \rightarrow -1 \leq x \leq 1$

$f(x)\in D_g \qquad \sqrt{1-x^2} \geq 0 \qquad 1-x^2 \geq 0 \qquad x^2 \leq 1 \rightarrow -1 \leq x \leq 1$

$D_g \rightarrow x \geq 0$

$D_{(f+g)\circ f} \rightarrow [-1,1]$

۹

الف) $f\left(\frac{3x+1}{x-2}\right) = 4x+5$

$\frac{3x+1}{x-2} = t \Rightarrow 3x+1 = tx-2t \Rightarrow 3x - tx = -2t-1$

$x(3-t) = -2t-1 \Rightarrow x = \frac{-2t-1}{3-t}$

$f(t) = 4\left(\frac{-2t-1}{3-t}\right)+5 = \frac{-8t-4}{3-t} + 5 = \frac{-8t-4+15-5t}{3-t} = \frac{-13t+11}{3-t} \Rightarrow f(x) = \frac{-13x+11}{3-x}$

$x \neq 3$ تعریف شود.

ب) $f\left(x+\frac{1}{x}\right) = x^3 + \frac{1}{x^3}$

$x+\frac{1}{x} = t \rightarrow \left(x+\frac{1}{x}\right)^3 = x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} \Rightarrow x^3 + \frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3x - \frac{3}{x}$

$x^3 + \frac{1}{x^3} = \underbrace{\left(x+\frac{1}{x}\right)}_{t}^3 - 3\underbrace{\left(x+\frac{1}{x}\right)}_{t} \Rightarrow f(t) = t^3 - 3t \Rightarrow f(x) = x^3 - 3x$

۱۰

$f(x) = x\sqrt{x}$

$g\circ f(x) \rightarrow$ انتقال طول نقاطی، در نمودار این تابع، محور xها را قطع می‌کند

$g\circ f(x) = g(f(x)) \rightarrow$ صفرهای $g(x)$ برابر x های ۱ یا $2\sqrt{2}$

if $g(f(x))$, $f(x)=1 \Rightarrow g(f(x)) = 0$ قرار دادن در ضابطه $\rightarrow$ if $f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow x = 1$

if $g(f(x))$, $f(x) = 2\sqrt{2} \Rightarrow g(f(x)) = 0 \rightarrow$ if $f(x) = 2\sqrt{2} \xrightarrow{f(x)=x\sqrt{x}} x\sqrt{x} = 2\sqrt{2} \Rightarrow x = 2$

$2-1 = 1$