

19, 70

نام و نام خانوادگی پاسخنامه تشریحی تکلیف شماره 9... کلاس
 نام و نام خانوادگی
 کلاس
 شماره
 تاریخ
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 تاریخ

$f(n) = \begin{cases} \cot \frac{\pi n}{4} & ; n \leq 1 \\ \sqrt{n^2 + 1} & ; n > 1 \end{cases}$ $\rightarrow f \circ f \left(\frac{1}{4} \right) \rightarrow f \left(\frac{1}{4} \right) = \cot \frac{\pi}{4} = 1 \rightarrow \cot \frac{\pi}{4} = \sqrt{2}$ $f(\sqrt{2}) = \sqrt{2^2 + 1} = \sqrt{5} = 2 = f \circ f \left(\frac{1}{4} \right) = 2$	1
الف) $f\left(\frac{1}{n}\right) = \sqrt{\frac{n-1}{n}}$, $g(n) = 2 \cos^2 n \rightarrow f(g(\frac{\pi}{4})) \rightarrow g(\frac{\pi}{4}) = 2 \cos^2 \frac{\pi}{4} = 2 \times \frac{1}{2} = 1$ $x=2 \rightarrow f\left(\frac{1}{2}\right) = \sqrt{\frac{2-1}{2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$ ب) $f(n) = [n]$, $g(n) = \frac{n}{1-n}$, $f \circ g(\sqrt{2}) = ? \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} = \frac{\sqrt{2}+1}{1-2} = \frac{\sqrt{2}+1}{-1} = -2-\sqrt{2}$ $f(-2-\sqrt{2}) = [-2-\sqrt{2}] = -2$	2
$f(n) = \sin n$, $g(n) = n \sqrt{1-n^2}$	3
الف) $f \circ g(n) = f(g(n)) = \{(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2})\}$ ب) $g \circ f(n) = g(f(n)) = \{\} = \emptyset$ ج) $f \circ f(n) = f(f(n)) = \{\} = \emptyset$ د) $g \circ g(n) = g(g(n)) = \{(1, n), (2, 4), (4, 1)\}$	4
$f = \{(1, 1), (\frac{1}{2}, 2), (\frac{1}{3}, 3), (1, 4)\}$, $g(n) = \{(1, 2), (2, 1), (\frac{1}{2}, 3), (\frac{1}{3}, 1)\}$	5

$(\frac{1}{2}, 2) \in f \circ g \rightarrow (\frac{1}{2}, 2) \in f(g(n)) \rightarrow a = 2$

$(1, 1) \in g \circ f \rightarrow (1, 1) \in g(f(n)) \rightarrow b = 1$

$$f(f(n)) = fn + r \rightarrow a(an+b) + b = fn + r \rightarrow \underbrace{a^2 n}_{a=r} + ab + b = fn + r$$

$$\xrightarrow{a=r} rb + b = r \rightarrow b = 1 \rightarrow f(n) = n + 1$$

$$g(rn + r) = rn - r \rightarrow g \circ f(-1) = ?$$

$$f(-1) = -1 \rightarrow g(-1) \Rightarrow rn + r = -1 \rightarrow g(-1) = -4 - r = -n$$

$$rn = -2$$

$$n = 2$$

$$f(n) = \sqrt{n+1}n, g(n) = \frac{1}{n^2 - 2n}$$

$$D_{g \circ f} = ? \Rightarrow D_{g \circ f} = \left\{ x \in D_f : f(n) \in D_g \right\} \rightarrow \left\{ R : f(n) \neq 0, \varepsilon \right\} \rightarrow D_{g \circ f} = \{ x : x \neq n \}$$

$$\rightarrow f(n) \neq 0 \rightarrow x > 0, x \neq n$$

$$f(n) = \sqrt{1-n^2}, g(n) = \sqrt{n}$$

$$D_{(f+g)(f(n))} \rightarrow D_{f+g} = \left\{ x \in D_f, f(n) \in D_{f+g} \right\} \rightarrow \left\{ 1 \leq x \leq 1, 0 \leq f(n) \leq 1 \right\} \rightarrow D_{(g+f)(f(n))} = [0, 1]$$

$$D_{f+g} = D_f \cap D_g \rightarrow D_{f+g} = 0 \leq n \leq 1$$

$$0 \leq \sqrt{1-n^2} \leq 1 \rightarrow 2x^2 \leq 1 \checkmark$$

$$0 \leq 1-n^2 \leq 1$$

$$-1 \leq -n^2 \leq 0$$

$$1 \geq n^2 \geq 0 \rightarrow 1 \geq x \geq 0$$

$$a) f\left(\frac{rn+1}{n-r}\right) = \varepsilon n + \Delta \rightarrow \frac{rn+1}{n-r} = t \rightarrow n = \frac{rt+1}{t-r} \rightarrow f(t) = f\left(\frac{rt+1}{t-r}\right) + \Delta = \frac{1rt-11}{t-r}$$

$$f(t) = \frac{1rt-11}{t-r} \rightarrow f(n) = \frac{1rn-11}{n-r}$$

$$b) f\left(n + \frac{1}{n}\right) = n^r + \frac{1}{n^r} \rightarrow n + \frac{1}{n} = t \rightarrow f\left(n + \frac{1}{n}\right) = \left(n + \frac{1}{n}\right)^r - r\left(n + \frac{1}{n}\right) \rightarrow f(t) = \varepsilon - rt$$

$$f(n) = n^r - rn$$

$$g \circ f(n) = g(f(n)) \xrightarrow{f(n)=1, r\sqrt{r}} g(n) = \cdot$$

$$f(n) = \begin{cases} 1 \rightarrow 1 = n\sqrt{n} \rightarrow \boxed{n=1} \\ r\sqrt{r} \rightarrow n\sqrt{n} = r\sqrt{r} \rightarrow \boxed{n=r} \end{cases} \rightarrow |r-1|=1$$