

تکلیف شماره ۹ - سید محمد زار یازدهم هنر

(19)

$$f(x) = \begin{cases} \cot \frac{\pi x}{2} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$$

$$f \circ f\left(\frac{1}{4}\right) = 1$$

$$f\left(\frac{1}{4}\right) = \cot\left(\frac{1}{4} \times \frac{\pi}{2}\right) = \cot\left(\frac{\pi}{4}\right) = \sqrt{1}$$

$$\sqrt{1+1} = \sqrt{2} = \sqrt{2} \quad f \circ f\left(\frac{1}{4}\right) = 1$$

(2)

الف) $f\left(\frac{1}{x}\right) = \sqrt{\frac{rx-1}{x^2}}$, $g(x) = r \cos^2 x$ $f \circ g\left(\frac{\pi}{4}\right) = 1$

$$g\left(\frac{\pi}{4}\right) = r \times \left(\frac{1}{2}\right)^2 = \frac{r}{4}$$

$$f\left(\frac{1}{4}\right) = \sqrt{\frac{1}{4} \times (r - \frac{1}{4})} = \sqrt{\frac{r}{4} \times \frac{3}{4}} = \frac{\sqrt{3r}}{2}$$

$$f(y) = \sqrt{\frac{r \times \frac{1}{y} - 1}{\left(\frac{1}{y}\right)^2}} = \sqrt{\frac{\frac{r}{y} - 1}{\frac{1}{y^2}}} = \sqrt{(y-r)y} \Rightarrow f(y) = \sqrt{(y-r)y}$$

ب) $f(x) = [x]$, $g(x) = \frac{\frac{1}{x^2} x}{1-x}$

$$f \circ g(\sqrt{2}) = f\left(\frac{1}{1-\sqrt{2}}\right) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{\sqrt{2}+2}{1-2} = -\sqrt{2}-2$$

$$f(-\sqrt{2}-2) = [-\sqrt{2}-2] = [-2, -1] = (-1)$$

(3)

$f(x) = \sin x$, $g(x) = x\sqrt{1-x^2}$, $f \circ g\left(\frac{\pi}{2}\right) = 1$

$$g\left(f\left(\frac{\pi}{2}\right)\right) \rightarrow g(1) \quad g(1) = 1 \times \sqrt{1-1} = 0$$

(4)

$f(x) = \{(1,5)(2,5)(3,12)(4,12)\}$, $g(x) = \{(1,4)(2,4)(3,8)(4,8)\}$

الف) $f \circ g(x) \rightarrow f(g(x))$

$$\begin{aligned} x=1 &\rightarrow g(x)=4 \rightarrow f(4)=12 \\ x=2 &\rightarrow g(x)=4 \rightarrow f(4)=12 \\ x=3 &\rightarrow g(x)=8 \rightarrow f(8)=12 \end{aligned}$$

$$x=4 \rightarrow g(x)=8 \rightarrow f(8)=12$$

$$f(g(x)) \rightarrow \{(1,12)(2,12)(3,12)(4,12)\}$$

ب) $g \circ f(x) \rightarrow x=1 \rightarrow f(x)=5 \rightarrow g(5) = \emptyset$

ج) $f \circ f(x) \rightarrow \emptyset$

د) $g \circ g(x) \rightarrow x=2 \rightarrow g(x)=4 \rightarrow g(4)=8$

$$x=3 \rightarrow g(x)=8 \rightarrow g(8)=12$$

$$x=4 \rightarrow g(x)=8 \rightarrow g(8)=12$$

$$x=1 \rightarrow g(x)=4 \rightarrow g(4)=8$$

$$g(g(x)) \rightarrow \{(1,8)(2,12)(3,12)(4,12)\}$$

(5)

$f = \{(1,1)(2,2)(3,3)(4,4)\}$

$g = \{(1,2)(2,1)(3,3)(4,4)\}$

$(a,b) = ?$ $(1,2) \in f \circ g$
 $(2,1) \in g \circ f$

$$f(g(x)) \begin{cases} 1 \rightarrow 2 \rightarrow 1 \\ 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 3 \rightarrow 3 \\ 4 \rightarrow 4 \rightarrow 4 \end{cases} \Rightarrow a=1$$

$g \circ f \begin{cases} 1 \rightarrow 1 \rightarrow 2 \\ 2 \rightarrow 2 \rightarrow 1 \end{cases} \quad b=2$

$(1,2) \rightarrow$

$$f(f(x)) = rx + r \quad f(f(x)) = f(ax+b) = a(ax+b) + b = a^2x + ab + b$$

$$g(rx+c) = rx - r \quad f(f(x)) = rx + r \Rightarrow a^2x + ab + b = rx + r \quad a^2 = 1 \quad a = \pm 1$$

$$g \circ f(-1) = ? \quad g(f(-1)) = r(2x+1) - r \quad ab + b = r \Rightarrow b(a+1) = r \xrightarrow{a=1} b = +1 \quad f(x) = rx + 1 \xrightarrow{a=-1} -1$$

$$f(x) = -rx - r \xrightarrow{a=-1} -1 \quad g(-1) = 4(-1) + 1 = -3 \quad f(-1) = -4(-1) - 1 = 3$$

$$g \circ f = ? \quad g \circ f = \{x \in D_f, f(x) \in D_g\}$$

$$f(x) = \sqrt{x+|x|} \quad g(x) = \frac{1}{x^2 - 4x}$$

$$\sqrt{x+|x|} = 0 \quad (x \leq 0) \quad \sqrt{x+|x|} = 1 \rightarrow (x=1) \quad D_{g \circ f} = \mathbb{R} - \{(-\infty, 0] \cup 1\}$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x} \quad (f+g) \circ f = ?$$

$$1-x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \quad D_f = [-1, 1]$$

$$(f+g) \circ f = [-1, 1]$$

$$f(f(x)) = \sqrt{1-(f(x))^2} \Rightarrow$$

$$1 - (f(x))^2 \geq 0 \Rightarrow 0 \leq (f(x))^2 \leq 1$$

$$f\left(\frac{rx+1}{x-r}\right) = \varepsilon x + \omega$$

$$\frac{rx+1}{x-r} = t \rightarrow xt - rt = rx + 1 \Rightarrow rt + 1 = xt - rx$$

$$rt + 1 = x(t-r) \Rightarrow x = \frac{rt+1}{t-r}$$

$$f(t) = \varepsilon x + \omega = \frac{(rt+1)\varepsilon}{t-r} + \omega \Rightarrow f(t) = \frac{\lambda t + \varepsilon}{t-r} + \omega = \frac{\lambda t + \varepsilon + \omega(t-r)}{t-r}$$

$$\frac{13t-11}{t-r} = f(t) \Rightarrow f(x) = \frac{13x-11}{x-r}$$

$$b) f\left(x + \frac{1}{x}\right) = x^r + \frac{1}{x^r} \quad x^r + \frac{1}{x^r} = \left(x + \frac{1}{x}\right)^r - r\left(x + \frac{1}{x}\right) \Rightarrow$$

$$t^r - rt = f(t) \Rightarrow f(x) = x^r - rx$$

$$g(x) = (b, 0) \cup (r\sqrt{r}, 0)$$

$$f(x) = x\sqrt{x}$$

$$D_f = x \geq 0$$

$$g(f(x)) = 0$$

$$x\sqrt{x} = r\sqrt{r} \Rightarrow x = r$$

$$f(x) = 1 \Rightarrow x\sqrt{x} = 1 \Rightarrow (x=1)$$

$$(1, r)$$

$$r-1 = r$$