

$$f(x) = \begin{cases} \cot \frac{\pi x}{\varepsilon} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases} \quad f\left(f\left(\frac{1}{\sqrt{2}}\right)\right) \xrightarrow{\frac{1}{\sqrt{2}} < 1} \cot \frac{1}{\sqrt{2}} \times \frac{\pi}{\varepsilon} = \frac{\pi}{4} \Rightarrow \cot \frac{\pi}{4} = \frac{\frac{\pi}{4}}{\frac{1}{\sqrt{2}}} = \sqrt{2}$$

$$\Rightarrow f(\sqrt{2}) = \sqrt{3 + 1} = 2$$

$$g\left(\frac{\pi}{\varepsilon}\right) = 2 \cos^2 \frac{\pi}{\varepsilon} = \frac{1}{\sqrt{2}} \Rightarrow f\left(g\left(\frac{1}{\sqrt{2}}\right)\right) = f\left(\frac{1}{\sqrt{2}}\right) = \sqrt{\frac{1}{2} - 1} = \frac{\sqrt{2}}{\sqrt{2}} \quad \text{الف) ۲-}$$

$$f(g(\sqrt{2})) \Rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1 - \sqrt{2}} \times \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{\sqrt{2} + 2}{1 - 2} = -\sqrt{2} - 2 \Rightarrow f(-\sqrt{2} - 2) = [-\sqrt{2} - 2] \quad \text{ب)}$$

$$\approx [-3, 4] = [-2]$$

$$g\left(f\left(\frac{\pi}{\varepsilon}\right)\right) \Rightarrow f\left(\frac{\pi}{\varepsilon}\right) = \sin \frac{\pi}{\varepsilon} = \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow g\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{2}}{\sqrt{2}} \sqrt{1 - \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^2} = \frac{\sqrt{2}}{\sqrt{2}} \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{4} = \frac{1}{2}$$

$$\text{الف) } f(g(x)) \Rightarrow 1) f \rightarrow 4 \rightarrow 5 \quad 2) 2 \rightarrow f \rightarrow 5 \quad 3) 4 \rightarrow 8 \rightarrow 12 \quad 4) 8 \rightarrow 16 \rightarrow 12 \quad \text{۴-}$$

$$= \{(f, 5), (2, 5), (4, 12), (8, 12)\}$$

$$\text{ب) } g(f(x)) = \emptyset$$

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$$\text{د) } g(g(x)) = 1) f \rightarrow 4 \rightarrow 8 \quad 2) 2 \rightarrow f \rightarrow 4 \quad 3) 4 \rightarrow 8 \rightarrow 16$$

$$= \{(f, 8), (2, 4), (4, 16)\}$$

$$(f, 2) \in f \circ g \Rightarrow f \rightarrow 3 \rightarrow 2 \Rightarrow a = f \quad \left. \begin{array}{l} (a, b) = (f, 5) \\ (f, 1) \in g \circ f \Rightarrow f \rightarrow 5 \rightarrow 1 \Rightarrow b = 5 \end{array} \right\}$$

$$f(x) = ax + b \Rightarrow f(f(x)) = a(ax + b) + b = a^2x + ba + b = f(x) \quad -4$$

$$\Rightarrow a^2 = x \Rightarrow a = \pm x, \quad x^2b + b = x \Rightarrow b = 1 \quad \therefore -x^2b + b = x \Rightarrow b = -x$$

$$g(x+x) = x^2 - x \rightarrow x^2 + x = t \Rightarrow x^2 = t - x \Rightarrow x = \frac{t-x}{x} \Rightarrow g(x) = \frac{x^2 - x}{x} = x$$

$$\Rightarrow g(x) = \frac{x^2}{x} - \frac{x}{x}, \quad f(x) = x^2 + 1 \quad \therefore f(x) = -x^2 - x$$

$$f(-1) = -1 \quad \therefore f(-1) = -1 \Rightarrow g(f(-1)) = g(-1) = \frac{-1^2}{-1} - \frac{-1}{-1} = -\frac{1}{-1} = \frac{1}{-1} = -1$$

$$D_f \Rightarrow x + |x| \geq 0 \rightarrow \left. \begin{array}{l} \text{if } x < 0 \Rightarrow f(x) = 0 \\ \text{if } x \geq 0 \Rightarrow f(x) = 2x \geq 0 \end{array} \right\} D_f = [0, +\infty) \quad \textcircled{1} \quad -v$$

$$D_g \Rightarrow x^2 - tx \neq 0 \Rightarrow x(x-t) \neq 0 \Rightarrow x \neq 0, x \neq t \rightarrow x > 0 \quad \textcircled{2}$$

$$D_{g \circ f} = \{x \in D_f : f(x) \in D_g\} \Rightarrow f(x) \in D_g \Rightarrow \sqrt{x+|x|} \neq 0, \sqrt{x+|x|} \neq t$$

$$\stackrel{\text{IC}}{\Rightarrow} x + |x| \neq 14 \rightarrow \text{if } x < 0 \Rightarrow f(x) = 0 \neq 14 \checkmark$$

$$\stackrel{\text{IC}}{\Rightarrow} D = (0, +\infty) - \{1\}$$

$$\textcircled{2} \text{ if } x > 0 \Rightarrow f(x) = 2x \Rightarrow 2x \neq 14 \Rightarrow x \neq 7$$

$$D_f \Rightarrow 1 - x^2 \geq 0 \Rightarrow x^2 \leq 1 \Rightarrow -1 \leq x \leq 1 \Rightarrow D_f = [-1, 1] \quad \textcircled{1}, \quad D_g = [0, +\infty) \quad -A$$

$$(f+g) \circ f = f(f(x)) + g(f(x)) \Rightarrow \sqrt{1-x^2} \geq 0 \rightarrow \text{دوره } (1R) \quad \textcircled{2}$$

در دامنه $E = [-1, 1]$ قرار می‌گیرد

$$\stackrel{\text{IC}}{\Rightarrow} D = [-1, 1]$$

$$\text{الف) } \frac{x^2 + 1}{x - 2} = t \rightarrow x^2 + 1 = tx \quad \text{if } t \Rightarrow (x^2 - tx) = -1 \Rightarrow x = \frac{t^2 + 1}{t - 2} \quad -A$$

$$\Rightarrow f(x) = f\left(\frac{t^2 + 1}{t - 2}\right) + 1 = \frac{1^2 x - 11}{x - 2}$$

$$\text{ب) } f\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3} \rightarrow \left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3x + \frac{3}{x} = \left(x^3 + \frac{1}{x^3}\right) + 3\left(x + \frac{1}{x}\right)$$

$$\Rightarrow x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) \quad \frac{x + \frac{1}{x} = t}{f(t) = t^3 - 3t} \Rightarrow f(x) = x^3 - 3x$$

$$\left. \begin{array}{l} g(x\sqrt{x}) \rightarrow x\sqrt{x} = x^{\frac{3}{2}} \Rightarrow x = 2 \rightarrow g(f(x)) = g(2) \\ x\sqrt{x} = 1 \Rightarrow x = 1 \rightarrow g(f(x)) = g(1) \end{array} \right\} [1, 2] \quad -10$$