

$$\frac{p}{r} = \cot \frac{\pi}{4} = \frac{\cos}{\sin} \frac{\pi}{4} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2}$$

$$f(\sqrt{p}) = \sqrt{(\sqrt{p})^2 + 1} = \sqrt{p+1}$$

$$\underbrace{f(-\sqrt{p}-r)}_{\approx -r, r} = [-r, r] = \overline{-r}$$

(ب)

$$f(g(\frac{\pi}{k}))$$

$$g\left(\frac{\pi}{r}\right) = r \cos^r \frac{\pi}{r} = \frac{1}{r}$$

$$f\left(\frac{1}{y}\right) = \sqrt{\frac{y}{x}} = \frac{\sqrt{y}}{x}$$

$$\underbrace{g\left(\frac{1}{f}\left(\frac{\pi}{f}\right)\right)}_{\sin \frac{\pi}{f} = \frac{\sqrt{p}}{p}}$$

$$g\left(\frac{\sqrt{y}}{y}\right) = \frac{\sqrt{y}}{y} \sqrt{\frac{1-\frac{y}{r}}{\frac{y}{r}}} = \frac{\sqrt{y}}{y} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{y}{y} = \frac{1}{y}$$

ب) $g \circ f(x) = g(f(x)) = \emptyset$

c) $f \circ f(x) = f(f(x)) = \emptyset$

$$\Rightarrow \text{gog}(A) = \{(f, 1), (2, 4), (4, 10)\}$$

$$\hookrightarrow g(g(x))$$

$$\mathcal{R}(g(x)) = \{(1,1), (3,4), (a,4), (b,4)\}$$

$$g(f(x)) = \{(x, y), (x, 1)\}$$

$$\Rightarrow (a, b) = \overline{(f, a)}$$

$$f(x) = ax + b \quad g(x) = px + q$$

$$f(f(x)) = px + q \Rightarrow f(f(x)) = a(ax + b) + b = apx + ab + b \Rightarrow$$

$$\begin{cases} +p \Rightarrow f(x) = px + 1 \Rightarrow f(-1) = -1 \\ \Rightarrow f(x) = -px - 1 \Rightarrow f(-1) = -1 \end{cases}$$

$$g(-1) = px - 1 = -1$$

$$px + q = -1 \Rightarrow px = -1 - q \Rightarrow x = -1$$

$$g \circ f(-1) = g(f(-1)) = ?$$

$$\begin{cases} +p \Rightarrow pb + b = q \Rightarrow b = 1 \\ -p \Rightarrow -pb + b = q \Rightarrow b = -1 \end{cases}$$

5

$$f(x) = \sqrt{x+1} \quad g(x) = \frac{1}{x^2 - x}$$

$$D_f \cap D_g = D_{f \circ g} = D_{g \circ f} = D_{f \circ g}$$

$$L \Rightarrow D_f = \mathbb{R}$$

$$x+1 \geq 0$$

$$g(f(x)) = \frac{1}{(f(x))^2 - f(x)}$$

$$\Rightarrow D_g = (0, 1) \cup (1, +\infty)$$

$$|x| = x \Rightarrow x = 1 \Rightarrow x = 1 \Rightarrow x \geq 0$$

$$|x| = -x \Rightarrow x+1 = 0 \Rightarrow x = -1 \Rightarrow x < 0$$

5

$$(f(x))^2 - f(x) \neq 0 \Rightarrow \sqrt{x+1} \neq 0 \Rightarrow x+1 \neq 0$$

$$L \Rightarrow x \leq 0 \Rightarrow f(x) = 0 \checkmark$$

$$L \Rightarrow f(x) = x \Rightarrow \sqrt{x+1} = x \Rightarrow x+1 = x^2$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x}$$

$$D_f \cap D_g = (f+g) \circ f = (f+g)(f(x)) = f(f(x)) + g(f(x))$$

$$f(f(x)) = \sqrt{1-(f(x))^2}$$

$$g(f(x)) = g(\sqrt{1-x^2}) = \sqrt{f(x)} = \sqrt{1-x^2}$$

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$$1-(f(x))^2 \geq 0 \Rightarrow 1-(\sqrt{1-x^2})^2 \geq 0 \Rightarrow 1-1+x^2 \geq 0 \Rightarrow x^2 \geq 0 \quad D_g = \mathbb{R}$$

$$\sqrt{1-x^2} \geq 0 \Rightarrow \checkmark \Rightarrow D_f = [-1, 1]$$

$$\Rightarrow D_{f+g} = [-1, 1]$$

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

$$f\left(\frac{px+1}{x-q}\right) = x^2 + \frac{1}{x^2}$$

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$$= \left(x + \frac{1}{x}\right)^2 - px + \frac{p}{x}$$

$$\begin{aligned} \frac{px+1}{x-q} = t &\Rightarrow tx - qt = px + 1 \Rightarrow tx - px = qt - 1 \Rightarrow x(t-p) = qt - 1 \\ \Rightarrow x(t-p) &= qt - 1 \Rightarrow x = \frac{qt-1}{t-p} \end{aligned}$$

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$$f(t) = t^2 - \frac{qt-1}{t-p} + \frac{p}{t-p} + \frac{1}{t-p}$$

$$\Rightarrow f(t) = \frac{t^2 + t + \frac{1}{t-p} - \frac{qt-1}{t-p}}{t-p} = \frac{t^2 + t - \frac{qt-1}{t-p}}{t-p} \Rightarrow f(x) = \frac{t^2 + t - \frac{qt-1}{t-p}}{t-p}$$

$$g(x) = 0 \Rightarrow x = 1 \quad \text{و} \quad \sqrt{2}$$

$$f(x) = x\sqrt{x}$$

$$g \circ f(x) = g(f(x)) = 0$$

$$f(x) = x\sqrt{x} \Rightarrow x = \sqrt{2} \Rightarrow \sqrt{2}\sqrt{\sqrt{2}} = 1 \Rightarrow \sqrt{2} - 1 = 1$$

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