

$$f \circ f\left(\frac{v}{w}\right) = f\left(f\left(\frac{v}{w}\right)\right) = ?$$

$$\frac{p}{r} = \cot \frac{\pi}{4} = \frac{\cos}{\sin} \frac{\pi}{4} = \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = \sqrt{2}$$

$$f(\sqrt{p}) = \sqrt{(\sqrt{p})^p + 1} = \sqrt{p}$$

$$\underbrace{f(g(\sqrt{x}))}$$

$$\frac{\sqrt{y}}{1-\sqrt{y}} \times \frac{1+\sqrt{y}}{1+\sqrt{y}} = \frac{\sqrt{y}+y}{\cancel{1-y}} = -\sqrt{y}-y$$

$$\underbrace{f(-\sqrt{p}-\epsilon)}_{\approx -w, \epsilon} = [-w, \epsilon] = \overline{-\epsilon}$$

(ب)

$$f(g(\frac{\pi}{k}))$$

$$g\left(\frac{\pi}{\nu}\right) = \nu \cos^{\nu} \frac{\pi}{\nu} = \frac{1}{\nu}$$

$$f\left(\frac{1}{r}\right) = \sqrt{\frac{r}{r}} = \frac{\sqrt{r}}{r}$$

$$\Rightarrow r = \sqrt{r}$$

$$g\left(f\left(\frac{1}{x}\right)\right)$$

$$\sin \frac{\pi}{4} = \frac{\sqrt{4}}{4}$$

$$g\left(\frac{\sqrt{y}}{y}\right) = \frac{\sqrt{y}}{y} \sqrt{1 - \frac{y}{y^2}} = \frac{\sqrt{y}}{y} \times \frac{\sqrt{y}}{y} = \frac{y}{y^2} = \frac{1}{y}$$

$$f \circ g(x) = f(g(x)) = \{(x, 5), (2, 5), (4, 12), (1, 12)\}$$

ب) $g \circ f(x) = g(f(x)) = \emptyset$

c) $f \circ f(x) = f(f(x)) = \emptyset$

$$\Rightarrow \text{gog}(K) = \{(f, 1), (v, 4), (4, 10)\}$$

$$\hookrightarrow g(g(x))$$

$$f(g(x)) = \{(1,1), (3,4), (a,2), (b,4)\}$$

$$g(f(x)) = \{(x, y), (x, 1)\}$$

$$\Rightarrow (a, b) = \gamma(f, a)$$

$$f(x) = ax + b \quad g(x) = px + q$$

$$f(f(x)) = px + q \Rightarrow f(f(x)) = a(ax + b) + b = apx + \underbrace{ab + b}_q \Rightarrow \begin{cases} ap = p \\ -q = -ab - b \end{cases}$$

$$\begin{cases} +p \Rightarrow f(x) = px + 1 \Rightarrow f(-1) = -1 \\ -p \Rightarrow f(x) = -px - 3 \Rightarrow f(-1) = -1 \end{cases}$$

$$g(-1) = px - 1 = -1$$

$$px + q = -1 \Rightarrow px = -1 - q = -1 - 1 = -2 \Rightarrow x = -2$$

$$g \circ f(-1) = g(f(-1)) = ?$$

$$\begin{cases} +p = 1 \Rightarrow pb + b = 1 \Rightarrow b = 1 \\ -p = -1 \Rightarrow -pb + b = 1 \Rightarrow b = -1 \end{cases}$$

$$f(x) = \sqrt{x+1} \quad g(x) = \frac{1}{x^2 - x}$$

$$D_f = \mathbb{R} \quad D_g = \mathbb{R} \setminus \{0, 1\}$$

$$L \Rightarrow D_f = \mathbb{R}$$

$$x+1 \geq 0$$

$$g(f(x)) = \frac{1}{(f(x))^2 - f(x)}$$

$$\Rightarrow D_g = (0, 1) \cup (1, +\infty)$$

$$|x| = x \Rightarrow x = 14 \Rightarrow x = 14 \text{ if } x \geq 0$$

$$|x| = -x \Rightarrow x + |x| = 0 \Rightarrow 14 \neq 0 \text{ if } x < 0$$

$$(f(x))^2 - f(x) \neq 0 \Rightarrow \sqrt{x+1} \neq 0 \Rightarrow x+1 \neq 0$$

$$L \Rightarrow x \leq 0 \Rightarrow f(x) = 0 \checkmark$$

$$L \Rightarrow f(x) = 1 \Rightarrow \sqrt{x+1} = 1 \Rightarrow x+1 = 1 \Rightarrow x = 0$$

$$f(x) = \sqrt{1-x^2} \quad g(x) = \sqrt{x}$$

$$D_f = \mathbb{R} \quad D_g = \mathbb{R} \setminus \{0, 1\}$$

$$f(f(x)) = \sqrt{1-(f(x))^2}$$

$$g(f(x)) = g(\sqrt{1-x^2}) = \sqrt{f(x)} = \sqrt{\sqrt{1-x^2}}$$

$$1-(f(x))^2 \geq 0 \Rightarrow 1-(\sqrt{1-x^2})^2 \geq 0 \Rightarrow 1-1+x^2 \geq 0 \Rightarrow x^2 \geq 0 \quad D_g = \mathbb{R}$$

$$\sqrt{1-x^2} \geq 0 \Rightarrow \checkmark \Rightarrow D_f = [-1, 1]$$

$$\Rightarrow D_{f \circ g} = [-1, 1]$$

$$f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

$$f\left(\frac{px+1}{x-q}\right) = px + q$$

$$(1)$$

$$= \left(x + \frac{1}{x}\right)^2 - px + \frac{p}{x}$$

$$\frac{px+1}{x-q} = t \Rightarrow tx - qt = px + 1 \Rightarrow tx - px = qt + 1 \Rightarrow x(t-p) = qt + 1 \Rightarrow x = \frac{qt+1}{t-p}$$

$$f(t) = t^2 - \frac{qt+1}{t-p} + \frac{p}{t-p} + q \Rightarrow f(t) = \frac{t^2 + t}{t-p} + q$$

$$f(x) = x^2 - px + \frac{p}{x}$$

$$\Rightarrow f(t) = \frac{t^2 + t + qt - q}{t-p} = \frac{t^2 + (q+1)t - q}{t-p} \Rightarrow f(x) = \frac{t^2 + (q+1)t - q}{t-p}$$

$$g(x) = 0 \Rightarrow x = 1 \quad \text{or} \quad x = -1$$

$$f(x) = x\sqrt{x}$$

$$g \circ f(x) = g(f(x)) = 0$$

$$f(x) = x\sqrt{x} \Rightarrow x = 1 \Rightarrow x = 1 \quad \text{or} \quad x = -1 \Rightarrow x = 1$$

$$\Rightarrow x - 1 = 1$$