

$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases}$
 $f \circ f\left(\frac{1}{2}\right) \rightarrow f\left(\frac{1}{2}\right) = \cot \frac{\pi}{8} \times \frac{1}{2} = \cot \frac{\pi}{4} = 1$
 $f(\sqrt{2}) = \sqrt{(\sqrt{2})^2 + 1} = \sqrt{3}$

الف) $f\left(\frac{1}{2}\right) = \sqrt{\frac{1-1}{2}} = 0$ $g(x) = 2 \cos^2 x$
 $f \circ g\left(\frac{\pi}{2}\right) \rightarrow g\left(\frac{\pi}{2}\right) = 2 \cos^2 \frac{\pi}{2} = 0$
 $f\left(\frac{1}{2}\right) = \sqrt{\frac{1-1}{2}} = 0$
 $f(x) = [x]$, $g(x) = \frac{x}{1-x}$
 $f \circ g(\sqrt{2}) \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}$
 $\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{-2-\sqrt{2}}{1-2} = \sqrt{2} + 2$
 $[2+\sqrt{2}] = 2$

$f(x) = \sin x$ $g(x) = x \sqrt{1-x^2}$ $g \circ f\left(\frac{\pi}{2}\right) \rightarrow f\left(\frac{\pi}{2}\right) = \sin \frac{\pi}{2} = 1$
 $g\left(\frac{\sqrt{2}}{2}\right) \rightarrow \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

الف) $f \circ g(x) = \begin{matrix} x \rightarrow y \rightarrow \omega \\ 2 \rightarrow 4 \rightarrow \delta \\ 4 \rightarrow 12 \rightarrow \pi \\ 12 \rightarrow 12 \rightarrow 12 \end{matrix}$ $f \circ g = \{(2, \delta), (4, \pi), (12, 12)\}$
 ب) $g \circ f(x) = \begin{matrix} x \rightarrow \delta \rightarrow \alpha \\ 4 \rightarrow \delta \rightarrow \alpha \\ 12 \rightarrow 12 \rightarrow \alpha \\ 12 \rightarrow 12 \rightarrow \alpha \end{matrix}$
 ج) $f \circ f(x) = \begin{matrix} x \rightarrow \delta \rightarrow \alpha \\ 4 \rightarrow \delta \rightarrow \alpha \\ 12 \rightarrow 12 \rightarrow \alpha \\ 12 \rightarrow 12 \rightarrow \alpha \end{matrix}$
 د) $g \circ g(x) = \begin{matrix} x \rightarrow 4 \rightarrow 12 \\ 2 \rightarrow 4 \rightarrow 12 \\ 4 \rightarrow 12 \rightarrow 12 \\ 12 \rightarrow 12 \rightarrow 12 \end{matrix}$

$f = \{(2, 1), (3, 2), (4, 3), (5, 4)\}$ $g = \{(1, 2), (2, 1), (3, 2), (4, 3)\}$
 $f, 2 \in f \circ g$ $(2, 1) \in g \circ f$
 $g \circ f = \begin{matrix} 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 2 \rightarrow 1 \\ 4 \rightarrow 3 \rightarrow 2 \\ 1 \rightarrow 4 \rightarrow 3 \end{matrix}$
 $f \circ g = \begin{matrix} 1 \rightarrow 2 \rightarrow 1 \\ 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 2 \rightarrow 1 \\ 4 \rightarrow 3 \rightarrow 2 \end{matrix}$

$a, b \in \omega \rightarrow (3, \omega)$

$$f(f(x)) = f(ax+b) = f(ax+b) = a(ax+b)+b = f^2x+b$$

$$g(x) = f^2x - 1$$

$$c(f^2x+b) + d = f^2x - 1 \rightarrow f^2cx + f^2c + d = f^2x - 1$$

$$f^2c = f^2 \rightarrow c = \frac{f^2}{f^2}$$

$$f^2c + d = f^2 \rightarrow \frac{f^2}{f^2} + d = f^2 \rightarrow d = f^2 - \frac{f^2}{f^2}$$

$$g(x) = \frac{f^2}{f^2}x - \frac{f^2}{f^2} \rightarrow g \circ f(-1) = f(-1) = -1 \quad g(-1) = \frac{f^2}{f^2} - \frac{f^2}{f^2} = -1$$

$$f(x) = \sqrt{x+|x|}$$

$$g(x) = \frac{1}{x^2 - f^2x}$$

$$g \circ f \rightarrow g(f(x)) = g(\sqrt{x+|x|})$$

$$\frac{1}{x+|x| - f^2\sqrt{x+|x|}} \rightarrow x+|x| \geq 0 \rightarrow \frac{D_f = \mathbb{R}}{0 \neq x} \rightarrow x \in D_f \rightarrow f(x) \in D_g$$

$$D_g = \mathbb{R} - \{f^2, 0\} \rightarrow \sqrt{x+|x|} = f^2 \rightarrow |x| + x = 14 \rightarrow x = 14 \rightarrow x = 14$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2}$$

$$g(x) = \sqrt{x}$$

$$(f+g) \rightarrow D_{f+g} = \{x \mid 1-x^2 \geq 0\} = [-1, 1]$$

$$(f+g) = [0, 1] \rightarrow f(x) \in D_{g+f} \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 0 \leq 1-x^2 \leq 1$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$a) f\left(\frac{f^2x+1}{x-f^2}\right) = f^2x+b$$

$$\frac{f^2x+1}{x-f^2} = f^2x+b \rightarrow x = -\frac{f^2+1}{f^2+1} = \frac{f^2+1}{f^2+1}$$

$$f\left(\frac{f^2x+1}{x-f^2}\right) = \frac{1x+f^2}{x-f^2} + d = \frac{1x+f^2+d(x-f^2)}{x-f^2}$$

$$\frac{1x-f^2}{x-f^2} = f(x)$$

$$b) f\left(x+\frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

$$\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \rightarrow x^2 + \frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2$$

$$x+\frac{1}{x} = t \rightarrow t^2 = x^2 + \frac{1}{x^2} + 2$$

$$f(x) = x^2 - f^2x$$

$$f(x) = x\sqrt{x}$$

$$g(f(x)) = 0 \rightarrow$$

$$f(x) = x\sqrt{x} \rightarrow x\sqrt{x} = x\sqrt{x} \rightarrow x = 1$$

$$|x-1| = 1$$

$$f(x) = 1 \rightarrow x\sqrt{x} = 1 \rightarrow x = 1$$