

$$f(x) = \begin{cases} \cot \frac{\pi x}{4} & ; x \leq 1 \\ \sqrt{x^2 + 1} & ; x > 1 \end{cases} \rightarrow f \circ f\left(\frac{1}{4}\right) \rightarrow f\left(\frac{1}{4}\right) = \cot \frac{\pi}{4} \times \frac{1}{4} = \cot \frac{\pi}{4} = \sqrt{3}$$

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$$f(\sqrt{3}) = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{4} = 2$$

$$f) f\left(\frac{1}{\alpha}\right) = \sqrt{\frac{\alpha^2 - 1}{\alpha^2}} \quad g(x) = 2 \cos^2 x$$

$$f \circ g\left(\frac{\pi}{4}\right) \rightarrow g\left(\frac{\pi}{4}\right) = 2 \cos^2 \frac{\pi}{4} = 1$$

$$f\left(\frac{1}{1}\right) = \sqrt{\frac{1^2 - 1}{1^2}} = \frac{\sqrt{0}}{1} = 0$$

$$\rightarrow f(x) = [x], \quad g(x) = \frac{x}{1-x}$$

$$f \circ g(\sqrt{2}) \rightarrow g(\sqrt{2}) = \frac{\sqrt{2}}{1-\sqrt{2}}$$

$$\frac{\sqrt{2}}{1-\sqrt{2}} \times \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{-2-\sqrt{2}}{1-2} = \sqrt{2} + 2 \rightarrow 1, 1, 2, 2, 3, 3, \dots$$

$$[-2, -1] = -2$$

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$$f(x) = \sin x \quad g(x) = x \sqrt{1-x^2} \quad g \circ f\left(\frac{\pi}{4}\right) \rightarrow f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$g\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{2} \sqrt{1 - \left(\frac{\sqrt{2}}{2}\right)^2} = \frac{\sqrt{2}}{2} \sqrt{1 - \frac{1}{2}} = \frac{\sqrt{2}}{2} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$$

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$$a) f \circ g(x) = \begin{matrix} x \rightarrow y \rightarrow \omega \\ 2 \rightarrow 4 \rightarrow \delta \\ 4 \rightarrow 12 \rightarrow \pi \\ 12 \rightarrow 12 \rightarrow \pi \end{matrix} \quad f \circ g = \{(2, \delta), (4, \delta), (12, \pi), (12, \pi)\}$$

$$\rightarrow g \circ f(x) = \begin{matrix} x \rightarrow \delta \rightarrow \alpha \\ 4 \rightarrow \delta \rightarrow \alpha \\ 12 \rightarrow 12 \rightarrow \alpha \\ 12 \rightarrow 12 \rightarrow \alpha \end{matrix}$$

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$$b) f \circ f(x) = \begin{matrix} x \rightarrow \delta \rightarrow \alpha \\ 4 \rightarrow \delta \rightarrow \alpha \\ 12 \rightarrow 12 \rightarrow \alpha \\ 12 \rightarrow 12 \rightarrow \alpha \end{matrix}$$

$$c) g \circ g(x) = \begin{matrix} x \rightarrow 4 \rightarrow 12 \\ 2 \rightarrow 4 \rightarrow 12 \\ 4 \rightarrow 12 \rightarrow 12 \\ 12 \rightarrow 12 \rightarrow 12 \end{matrix} \quad g \circ g(x) = \{(2, 12), (4, 12), (12, 12), (12, 12)\}$$

$$f = \{(2, 1), (3, 2), (4, 3), (1, 4)\}$$

$$g = \{(1, 2), (3, 1), (4, 3), (2, 1)\}$$

$$f, g \in f \circ g \quad (2, 1) \in g \circ f$$

$$f \circ g = \begin{matrix} 1 \rightarrow 2 \rightarrow 1 \\ 3 \rightarrow 1 \rightarrow 1 \\ 4 \rightarrow 3 \rightarrow 2 \\ 2 \rightarrow 1 \rightarrow 1 \end{matrix} \quad (2, 1) = (2, 1)$$

$$g \circ f = \begin{matrix} 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 2 \rightarrow 2 \\ 4 \rightarrow 3 \rightarrow 1 \\ 1 \rightarrow 4 \rightarrow 2 \end{matrix}$$

$$a \rightarrow 3 \rightarrow 2 \quad a = 3$$

$$b \rightarrow 1 \rightarrow 1$$

$$f \circ g = \{(1, 1), (3, 1), (4, 1), (2, 1)\}$$

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$$a, b \in f \rightarrow (3, 2)$$

$$f(f(x)) = f(ax+b) = f(ax+b) = a(ax+b)+b = f^2x+f^2$$

$$g(x) = f^2x - f^2$$

$$ax+b = f^2x+f^2$$

$$c(ax+b) + d = f^2x - f^2 \rightarrow f^2cx + f^2c + d = f^2x - f^2$$

$$a^2 = f^2 \rightarrow a = \pm f \rightarrow f(x) = f^2x - f^2$$

$$f(x) = f^2x + 1$$

$$f^2c + d = f^2 \rightarrow \frac{f^2}{f} + d = f^2 \rightarrow d = f^2 - \frac{f^2}{f}$$

$$g(x) = \frac{f^2}{f}x - \frac{f^2}{f} \rightarrow g \circ f(-1) = f(-1) = -1 \quad g(-1) = \frac{-f^2}{f} - \frac{f^2}{f} = -\frac{2f^2}{f} = -2$$

$$f(x) = \sqrt{x+|x|}$$

$$g(x) = \frac{1}{x^2 - f^2x}$$

$$g \circ f \rightarrow g(f(x)) = g(\sqrt{x+|x|})$$

$$\frac{1}{x+|x| - f^2\sqrt{x+|x|}}$$

$$\rightarrow x+|x| \geq 0 \rightarrow \frac{1}{x+|x|} \geq 0 \rightarrow x \in D_f \rightarrow f(x) \in D_g$$

$$D_g = \mathbb{R} - \{f^2, 0\} \rightarrow \sqrt{x+|x|} = f^2 \rightarrow |x| + x = f^4 \rightarrow x = f^4 \rightarrow x = 1$$

$$D_{g \circ f} = (0, +\infty) - \{1\}$$

$$f(x) = \sqrt{1-x^2}$$

$$g(x) = \sqrt{x}$$

$$(f+g) \rightarrow D_f = 1-x^2 \geq 0 \rightarrow -1 \leq x \leq 1$$

$$(f+g) = [0, 1] \rightarrow f(x) \in D_{g+f} \rightarrow 0 \leq \sqrt{1-x^2} \leq 1 \rightarrow 0 \leq 1-x^2 \leq 1$$

$$\rightarrow x^2 \leq 1 \rightarrow -1 \leq x \leq 1$$

$$D_{(f+g) \circ f} = [-1, 1]$$

$$a) f\left(\frac{f^2x+1}{x-f^2}\right) = f^2x + 1$$

$$\frac{f^2x+1}{x-f^2} = f^2x+1 \rightarrow x = -\frac{f^2+1}{f^2+1} = \frac{f^2+1}{f^2+1}$$

$$f\left(\frac{f^2x+1}{x-f^2}\right) = \frac{1x+1}{x-f^2} + 1 = \frac{1x+1+f^2x-f^2}{x-f^2} = \frac{(1+f^2)x+1-f^2}{x-f^2}$$

$$\frac{1x+1}{x-f^2} = f(x)$$

$$b) f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$$

$$\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 \rightarrow x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$x + \frac{1}{x} = t \rightarrow t^2 = x^2 + \frac{1}{x^2} + 2$$

$$f(x) = x^2 - f^2x$$

$$f(x) = x\sqrt{x}$$

$$g(f(x)) = 0 \rightarrow$$

$$f(x) = x\sqrt{x} \rightarrow x\sqrt{x} = f^2x \rightarrow x = f^2$$

$$x\sqrt{x} = 1 \rightarrow x = 1$$

$$|x-1| = 1$$